Math 342 —Exam 1 review

Your exam in class on October 18 will contain about 5 multi-part problems. It will cover material from Homework 0 to Homework 4. In the textbook, this is material spanning sections 1.1 through 3.4 (the material up through the binomial distribution), though we've skipped some sections in that range which will not be part of the exam. There will be no material related to R or Monte Carlo simulation on the exam. The problems below give you a sampling of some problems like those that will appear on the exam, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also suggested exercises that have been given as part of each homework, most of which have answers at the back of the textbook. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet. You'll be allowed to use a scientific calculator with no graphing or calculus functionality, but the problems will be written so that a calculator is not required.

Problem 1. Suppose that A, B, and C are events in an experiment, with C and $A \cup B$ mutually exclusive and

$$P(AB^c) = 1/6, \quad P(BA^c) = 1/4, \quad P(AB) = 1/12, \quad P(C) = 5/12$$

Find the probability of each of the following:

a. A

b. at least one of A or B occurs

c. exactly one of the three events occurs

d. all three events occur

e. at least one of the three events occurs

Problem 2. A symphony orchestra has in its repertoire 30 Haydn pieces, 15 modern pieces, and 9 Beethoven pieces. A program consists of three different pieces from the repertoire. Suppose we choose a program at random. Find the probability that the program has

a. two modern pieces

b. more than one piece of the same type

c. a Hayden piece first, followed by 2 modern pieces

Problem 3. A bag of Scrabble tiles contains two of each of the letters R, A, N, D, O, and M for a total of 12 tiles. Six tiles are picked without replacement and placed left to right on a Scrabble rack. Find the probability that you:

a. spell R-A-N-D-O-M from left to right

b. pick both R's

c. pick no M's

Problem 4. Every Saturday afternoon Carmen plays golf with probability 0.3 or plays squash with probability 0.7. After the golf game, she goes out for a massage with probability 0.55, and after the squash game, she goes out for a massage with probability 0.2.

a. Find the probability that she will go out for a massage.

b. If she goes out for a massage, what is the probability that she played golf?

Problem 5. There are three coins in a box. One is two-headed, one is fair, and one is biased to come up heads with probability 0.75. A coin is selected at random, flipped, and shows heads. What is the probability that it was the two-headed coin?

Problem 6. Suppose A, B, C are independent events with respective probabilities 1/6, 1/4, and 1/2. Find the probability that

a. at least one of the events occurs

b. A does not occur, given that both B and C occur

c. A and B occur, given that A or B occur

Problem 7. A coin has heads probability 1/3.

a. Find the probability that among 7 tosses of the coin

    1. no heads appear
    2. exactly 3 heads appear
    3. at least 5 heads appear

b. Suppose 5 people each make 7 tosses of the coin. Find the probability that at least 3 of them get no heads.

Problem 8. For the following situations determine whether the binomial distribution is a reasonable model for the given random variable. If so, state its parameters. If not, explain why.

a. Grant believes there is a 40 percent chance of rain tomorrow. Let X indicate the presence or absence of rain tomorrow.

b. Dana and Curtis are playing a strategy game. They are equally likely to win, and play 10 matches. Let X denote the number of Dana's wins out of those matches.

c. Eddie spends his free afternoons watching ship traffic in the harbor. Each hour about 4 large ships arrive to dock at the port. Let X be the number of large ships which arrive in the next hour.

d. Marilyn is playing a board game where income per turn is generated by rolling a standard six-sided die and multiplying the result by 100. Let X be the income earned on a turn.

Problem 9. Let $X_1 \sim \text{Ber}(0.2)$ and$X_2 \sim \text{Ber}(0.7)$ be independent random variables. Find the probability mass function of $Y = X_1 + X_2$. What is the probability mass function of Y if X_1 and X_2 have the same parameter p?