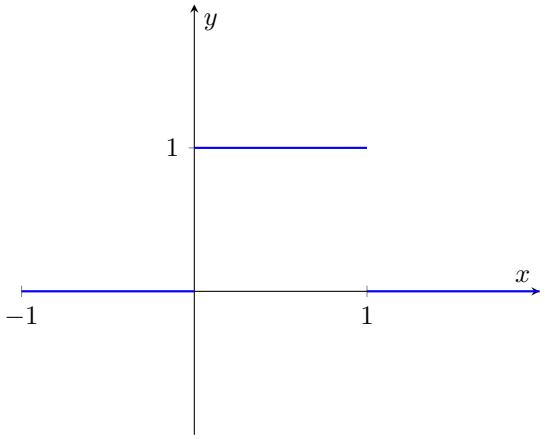
Math 342 —Probability density functions

Problem 1. Consider the piecewise defined function $f : \mathbb{R} \to \mathbb{R}$ given by.

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Its plot is given here:



Compute each of the following using just the elementary fact that the area of a rectangle is bh where b denotes the length of the base and h denotes the length of the height.

a. $\int_0^{1/4} f(t)\,dt$

b. $\int_0^{1/2} f(t)\,dt$

c. $\int_0^{2/3} f(t)\,dt$

d. $\int_{1/3}^{2/3} f(t)\,dt$

e. $\int_2^3 f(t)\,dt$

f. $\int_{1/2}^2 f(t)\,dt$

g. $\int_{-\infty}^0 f(t)\,dt$

h. $\int_{-\infty}^1 f(t)\,dt$

i. $\int_{-\infty}^{\infty} f(t)\,dt$

j. $\int_0^x f(t)\,dt$ if $0 < x < 1$ (your answer will be in terms of x)

k. $\int_0^x f(t)\,dt$ if $x \geq 1$

l. $\int_0^x f(t)\,dt$ if $x \leq 0$

Problem 2. Let X be a random variable with density f given by

$$f(x) = \begin{cases} cx^2 & -2 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

a. What value of c makes it so that $\int_{-\infty}^{\infty} f(t)\,dt = 1$?

b. For each of the following definite integrals, draw a plot of $f(x)$, shade in the area represented by the integral, and then compute a value for the integral/area.

   1. $\int_{-\infty}^{1} f(t)\,dt$
   2. $\int_{1}^{\infty} f(t)\,dt$ (can you use your answer to the previous integral when computing this?)
   3. $\int_{-1}^{2} f(t)\,dt$

c. What probabilities do the previous integrals represent?

Problem 3. Let X be a random variable with density f given by

$$f(x) = \begin{cases} 1-x & 0 \le x < 1 \\ cx^2 & 1 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

a. What value of c makes it so that $\int_{-\infty}^{\infty} f(t)\,dt = 1$?

b. For each of the following definite integrals, draw a plot of $f(x)$, shade in the area represented by the integral, and then compute a value for the integral/area.

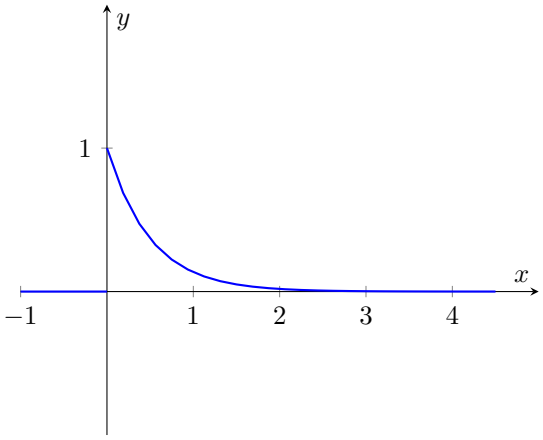
   1. $\int_{-\infty}^{1} f(t)\,dt$
   2. $\int_{1}^{\infty} f(t)\,dt$ (can you use your answer to the previous integral when computing this?)
   3. $\int_{1/2}^{3/2} f(t)\,dt$

c. What probabilities do the previous integrals represent?

Problem 4. Let X be a random variable whose density is given by

$$f(x) = \begin{cases} ce^{-2x} & x \ge 0 \\ 0 & x < 0. \end{cases}$$

The plot of f is given here:



a. Find c so that $\int_{-\infty}^{\infty} f(t)\,dt = 1$

b. Compute:

   1. $P(X < 1)$
   2. $P(X = 1)$
   3. $P(1 < X < 2)$
   4. $P(X > 2)$
   5. $P(X \le x)$ for an arbitrary positive number x
   6. $P(X \le x)$ for an arbitrary negative number x