Math 342 —Cumulative distribution functions

Problem 1. Suppose X is a continuous random variable with density

$$f(x) = \begin{cases} c & -4 \leq x \leq 7 \\ 0 & \text{otherwise.} \end{cases}$$

a. Draw the graph of f and find c.

b. Find an expression for $F(x) = P(X \leq x)$ in terms of x when

   1. $x < -4$
   2. $-4 \leq x \leq 7$
   3. $x > 7$

c. Use $F(x)$ to find the following probabilities. Do not do any integration.

   1. $P(-3 < X < 1)$
   2. $P(X \geq 1.5)$
   3. $P(-5 \leq X \leq 5)$

Problem 2. Consider the continuous random variable X whose density is given by

$$f(x) = \begin{cases} cx^2 & -1 < x \leq 0 \\ x & 0 < x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

a. Draw the graph of f and find c.

b. Find an expression for $F(x) = P(X \leq x)$ in terms of x when

   1. $x \leq -1$
   2. $-1 < x \leq 0$
   3. $0 < x \leq 1$
   4. $x > 1$

c. Use $F(x)$ to find the following probabilities. Do not do any integration.

   1. $P(-0.25 \leq X \leq 0.75)$
   2. $P(X \leq -0.5)$
   3. $P(X > 0.5)$

Problem 3. Suppose X is a continuous random variable with cdf given by

$$F(x) = P(X \leq x) = \begin{cases} 0 & x \leq 0, \\ \frac{x^4}{16} & 0 < x < 2, \\ 1 & x \geq 2. \end{cases}$$

a. Draw a plot of $F(x)$.

b. Find $f(x)$ and draw a plot on a separate set of axes.

c. How is the area under f related to F?

d. Use $F(x)$ to find the following probabilities. Do not do any integration.

   1. $P(X > 1)$
   2. $P(1 \leq X \leq 2)$
   3. $P(1/2 < X < 10)$