Math 342 —Exponential distribution

Problem 1. Let $X \sim \text{Exp}(5)$ be the time in hours until you receive a spam email message.

a. Find the probability it takes less than 2 hours to receive a spam message.

b. Find the probability it takes between .75 and 1.3 hours to receive a spam message.

c. Find the expected time in hours until you receive a spam message.

d. Find the expected time in minutes until you receive a spam message.

e. Find the average rate, in messages per hour, at which you receive spam messages.

f. Find the average rate, in messages per minute, at which you receive spam messages.

Problem 2. Suppose that phone calls arrive at a call center, which opens at 8:00 a.m., according to a Poisson process with an average rate of 1 call every 15 minutes. Let X denote the amount of time elapsed since 8:00 a.m. until the first phone call.

a. What is the value of λ if using units of hours? Minutes?

b. Use units of minutes to find the probability that the first phone call occurs

   1. after 8:30 a.m.
   2. before 8:20 a.m.
   3. between 8:15 and 8:45 a.m.

c. How do your answers change if you convert your work to hours?

d. Suppose it's now 9:00 a.m. and no phone calls have come. Given this, what is the probability that you receive no calls in the next 15 minutes?

e. Suppose you want to sleep in a little and arrive at the call center after 8:00 a.m., knowing that it's unlikely for the first call of the day to come in right when the center opens. You don't want to miss the first call however. How late can you be so that you're 90% sure you get in before the first call?

f. Find $E[X]$ and find the expected time of the first call.

g. Suppose I tell you that on average there's 45 minutes between phone calls. What is the value of λ in this case? What is $E[X]$?