

Math 342 —Double integrals

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a given function where

$$\text{supp}(f) = \{(x, y) \in \mathbb{R}^2 : 0 < x < 1, 0 < y < 1\},$$

and consider the following given sets:

$$A = \{(x, y) \in \mathbb{R}^2 : x < y\}$$

$$B = \{(x, y) \in \mathbb{R}^2 : x + y < 1\}$$

$$C = \{(x, y) \in \mathbb{R}^2 : y > 1/4\}$$

$$D = \{(x, y) \in \mathbb{R}^2 : y < x^3\}$$

$$E = \{(x, y) \in \mathbb{R}^2 : y > x/2\}.$$

Problem 1. For each set:

- Sketch its intersection with $\text{supp}(f)$ on the xy -plane.
- Set up a double integral that gives the volume under f and above the set in two ways: with $dA = dydx$ and with $dA = dxdy$.

Problem 2. Suppose $f(x, y) = cxy$ on its support.

- Find the value of c so that the total volume under f is 1.
- Compute $\iint_E f(x, y) dA$ where E is the set given above.