Math 342 —Joint densities

Problem 1. Let X and Y have joint density given by

$$f(x,y) = \begin{cases} 6(x-y) & 0 \le y \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Express each of the following probabilities as a double integral. Make sure you can compute these by hand, but save that for later. You may use Wolfram Alpha to check your answer when you do so.

   a. $P(X \le 1/2)$

   b. $P(Y < X^3)$

   c. $P(X + Y < 1)$

Problem 2. Compute $E[X^2 Y]$ using the joint density from Problem 1.

Problem 3. Let X and Y have joint density given by

$$f(x,y) = \begin{cases} 2e^{-(x+y)} & 0 < y < x < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Express each of the following probabilities as a double integral.

   a. $P(Y > 2)$

   b. $P(2Y > X)$

   c. $P(X + Y > 2)$