

Math 342 —Marginals and independence

Problem 1. Let X and Y have joint density given by

$$f(x, y) = \begin{cases} 6(x - y) & 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- Find the marginal density of X . Make sure to give a piecewise definition.
- Find the marginal density of Y . Make sure to give a piecewise definition.
- Are X and Y independent?

Problem 2. Answer the questions of Problem 1 using the joint density

$$f(x, y) = \begin{cases} 2e^{-(x+y)} & 0 < x < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Problem 3. Suppose Xavier and Yolanda are planning to meet at a coffee shop for lunch. Xavier will arrive at some time uniformly distributed between 1:00 and 1:30 p.m, and Yolanda will arrive at some time uniformly distributed between 1:00 and 1:20. Their arrival times are independent.

- What is the probability that Xavier arrives at least 10 minutes after Yolanda?
- What is the probability that they arrive within 5 minutes of each other?