Math 342 —Normal distribution

Problem 1. Let $X \sim N(-4, 25)$. Find the approximate value of the following probabilities using the 68-95-99.7 rule.

a. $P(-14 < X < 6)$

b. $P(X > 1)$

c. $P(-9 < X < 6)$

Problem 2. Let $X \sim N(-4, 25)$. Find the following probabilities with R.

a. $P(|X| < 2)$

b. $P(e^X < 1)$

c. $P(X^2 > 3)$

Problem 3. The length of human pregnancy is normally distributed with mean $\mu = 270$ and standard deviation $\sigma = 10$ days. Find the probability that a random pregnancy takes longer than 290 days or less than 240 days.

Problem 4. Babies' birth weights are normally distributed with mean 120 ounces and standard deviation 20 ounces.

a. Use the 68-95-99.7 to find the probability a randomly selected baby's birth weight is greater than 160 ounces.

b. *Low birth weight* is an important indicator of a newborn baby's future health. One definition of having low birth weight is that a baby weighs less than the 5th percentile of the weight distribution. State the threshold in ounces of what is considered a low birth weight.

c. Suppose *very low birth weight* is defined as a weight less than the 3rd percentile of the weight distribution. Find the conditional probability that a baby with low birth weight has very low birth weight.

Problem 5. Let $X \sim N(\mu, \sigma^2)$. Suppose $a, b \in \mathbb{R}$ are given constants with $a \neq 0$. Let $Y = aX + b$.

a. Express $F_Y(x) = P(Y \leq x)$, the CDF of X, in terms of F_X, the CDF of X, using algebraic manipulation.

b. Compute $F_Y'(x)$ to get the density of Y. What can you conclude about the distribution of Y?