Math 342 —MGFs and Sums of Normal random variables

Problem 1. Let $X \sim N(\mu, \sigma^2)$. Use the moment generating function of $Z \sim N(0, 1)$ to find the moment generating function of X. *Hint*: remember that $Z = (X - \mu)/\sigma$.

Problem 2. Let $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ be independent normal random variables.

a. Find the moment generating function of $X + Y$.

b. Explain why $X + Y$ is normally distributed and give its mean and variance.

Problem 3. Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$ be i.i.d. normal random variables. Let $S_n = X_1 + \cdots + X_n$. Explain why S_n/n is normally distributed and give its mean and variance.

Problem 4. Before paying at a fruit stand, your fruit is weighed on a scale that is a bit unreliable. The weight output by the scale is a random variable $X = w + M$ where w is the true weight of your fruit and $M \sim N(0, 4)$ is the measurement error (in ounces).

a. If you weight your fruit, what is the probability that the weight output by the scale is within 1 ounce of the true weight w?

b. Suppose you weigh your fruit 5 times and take the average of the resulting weights. What is the probability that the average is within 1 ounce of the true weight w?