Math 342 —Conditional densities

Problem 1. Let X and Y have joint density

$$f(x, y) = \begin{cases} 4e^{-2x} & 0 < y < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

a. Make a sketch of $\text{supp}(f)$.

b. Find the marginal density of X.

c. Write an expression for $f_{Y|X}(y \mid x)$.

d. Write an expression for $f_{Y|X}(y \mid 5)$.

e. Find $P(Y > 1 \mid X = 5)$ by computing an integral.

Problem 2. Suppose you are given the conditional density $f_{Y|X}(y \mid x)$ and marginal densities $f_X(x)$ and $f_Y(y)$. Try to derive a formula for the conditional density $f_{X|Y}(x \mid y)$ in terms of these three given densities. What do you think this formula should be called?

Problem 3. Suppose Alice picks a random number X uniformly distributed in the interval $(0, 10)$. Then if Alice's number is $X = x$, Bob picks a number Y uniformly distributed in the interval $(0, x)$. This is sometimes called a *hierarchical model.*

a. Suppose we know Alice picked the number 6. Find the probability that Bob's number is greater than 4 by first doing the following.

    1. State the marginal density $f_X(x)$ of X.

    2. State the conditional density $f_{Y|X}(y \mid x)$ of Y given $X = x$. *Note no calculation is necessary.*

b. Suppose we only saw the second step of the experiment. That is, we saw that Bob picked 1. The find the probability that Alice's number is greater than 9 by first doing the following.

    1. State the joint density of X and Y. *Note no calculation is necessary.*

    2. Find the marginal density $f_Y(y)$ of Y.

    3. Use your answer to Problem 2 to state the conditional density $f_{X|Y}(x \mid 1)$ of X given $Y = 1$.