Math 342 —Law of Total Expectation

Problem 1. Alice will harvest a random number $T \sim \text{Pois}(55)$ of tomatoes from her vegetable garden over the course of a season, distributed. Suppose that each tomato, independent of others, is defective with probability 0.4. Let X be the number of defective tomatoes.

  a. State the conditional distribution of X given $T = n$.

  b. Find $E[X \mid T]$.

  c. Find $E[X]$.

Problem 2. Suppose that the number of emails you get in a day is $N \sim \text{Pois}(20)$. Moreover, the time it takes you to read and respond to each is random and independent, from email to email and independent of the number of emails you get. Suppose that the time for each email is exponentially distributed with a mean of 15 minutes. Let $X_1, \ldots, X_N$ be the time spent for each of these N emails, and let T denote the total time in hours spent on emails in a given day.

  a. Find $E[T \mid N = n]$.

  b. Find $E[T \mid N]$.

  c. Find $E[T]$.

Problem 3. Suppose that X and Y have joint density

$$f(x, y) = \begin{cases} e^{-y} & 0 < x < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

  a. Find $E[Y \mid X = x]$.

  b. Find $E[Y \mid X]$.

  c. Find $E[Y]$.