Math 342 — Exam 3 review

Your exam during the self-scheduled exam period will contain about 5 multi-part problems. It will cover material from Homework 8 to Homework 10, along with material on the Law of Large Numbers and Central Limit Theorem but not the proof of the CLT. In the textbook, this is material spanning sections 6.5 through 10.5, though we've skipped some material in that range which will not be part of the exam. There will be no material related to R on the exam. The problems below give you a sampling of some problems like those that will appear on the exam, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also suggested exercises that have been given as part of each homework, most of which have answers at the back of the textbook. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet. You'll be allowed to use a scientific calculator with no graphing or calculus functionality, but the problems will be written so that a calculator is not required.

Problem 1. Let X and Y have joint density given by

$$f(x,y) = \begin{cases} cx & 0 < y < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

a. Find c.

b. Set up a double integral for $P(X + Y \leq 1)$.

c. Find the marginal density of X.

d. Find the marginal density of Y.

e. Are X and Y independent?

f. Find $P(X < 0.75 \mid Y = 0.25)$.

g. Find $P(Y > 0.2 \mid X = 0.75)$.

h. Find $E[Y \mid X]$

Problem 2. Repeat the previous problem with the joint density

$$f(x,y) = \begin{cases} cxy & 0 < x < y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Problem 3. Suppose that X is uniform on the interval $(0, 3)$ and that $Y \mid X = x$ is uniform on the interval $(0, x^2)$. Find the following

a. $P(Y < 4 \mid X = x)$ when $0 < x < 3$

b. $P(Y < 4)$

c. $E[Y \mid X = x]$ when $0 < x < 3$

d. $E[Y]$

Problem 4. Consider a random variable X with probability mass function given by

$$P(X = x) = \begin{cases} 1/5 & x = 10 \\ 3/10 & x = 20 \\ 1/2 & x = 30. \end{cases}$$

Given $X = x$, we flip a fair coin x times and let Y count the number of heads that result.

a. Give the conditional distribution of Y given $X = x$. Make sure to specify parameters.

b. Find $E[Y|X]$.

c. Find $E[Y]$.

Problem 5. Someone receives on average 5 text messages per hour. Let X denote the time until they receive a message.

a. What distribution should be used to model X? What parameter(s) does it have?

b. Find the expected time until receiving a message.

c. Find the probability that it takes between 10 and 20 minutes before receiving a message.

d. Find the probability that it takes less 30 minutes to receive a message.

e. Suppose no message has been received after 30 minutes. Find the probability that still no message has been received after 5 more minutes.

Problem 6. Suppose that X is a normal random variable with mean 10 and variance σ^2. Suppose $P(X > 15) = 0.16$. Use the 68-95-99.7 rule to estimate the following quantities.

a. σ^2

b. $P(X < 5)$

c. $P(0 < X < 20)$

d. $P(X > 25)$

Problem 7. Suppose X and Y are i.i.d. normally distributed random variables random variables with mean μ and variance σ^2. Give the distribution, including any parameters, of the following random variables.

a. $4X + 7$

b. $-3Y + 5$

c. $X + Y$

d. $X - Y$

Problem 8. Explain how to use Monte Carlo simulation to approximate the following integrals.

a. $\int_0^1 \sin(x^{-x})\, dx$

b. $\int_1^{10} \sin(x^{-x})\, dx$

c. $\int_0^\infty \sin(x^{-x})\, dx$

Problem 9. Use the Central Limit Theorem to give the approximate distribution of the random variable X in the following scenarios.

a. The local farm packs its tomatoes in crates. Individual tomatoes have mean weight of 10 ounces and standard deviation 3 ounces. Let X be the weight of a crate of 50 tomatoes.

b. For one \$1 red bet, let G be the casino's gain. Then $P(G = 1) = 20/38$ and $P(G = -1) = 18/38$. Suppose in 1 day, 1000 red bets are placed. Let X be the casino's total gain.

c. Let X be the average of 100 random numbers sampled from the distribution with density $f(x) = 6x(1 - x)$ supported on the interval $(0, 1)$.