

Math 342 — Conditional probability

Problem 1. In a certain town 30% of people own cats. Of the cat owners, 20% own dogs, while town-wide, 40% of people own dogs. What is the probability a person owns both a cat and a dog? What is the probability a dog owner owns a cat?

Problem 2. An urn contains 6 white and 9 green balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 green?

Problem 3. A bag contains 15 tiles from the game Scrabble; 5 have the letter M, 5 have the letter N, and 5 have the letter O. Four tiles are chosen at random, one at a time, without replacement. What is the probability that they spell out M-O-M-O (in that order)?

Problem 4. For each of the following conditions, find a simplified expression for $P(A | B)$.

- a. $A = B$
- b. $B \subseteq A$
- c. $A \subseteq B$
- d. A and B are mutually exclusive events.

Problem 5. Derive the following formula by simplifying the left hand side until you get the right hand side:

$$P(A | B) + P(A^c | B) = 1.$$

Problem 6 (Bonus challenge problem). A recent college graduate is planning to take the first three actuarial examinations in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the conditional probability that she passes the second one is 0.8, and if she passes both the first and the second exams, then the conditional probability that she passes the third exam is 0.7.

- a. What is the probability that she passes all three exams?
- b. Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?