

## Math 342 — Law of total probability and Bayes' formula

**Problem 1.** Amy has two bags of candy. The first bag contains two packs of M&Ms and three packs of Gummi Bears. The second bag contains four packs of M&Ms and two packs of Gummi Bears. Amy chooses a bag at random—the first chosen with probability  $1/4$  and the second bag chosen with probability  $3/4$ — and then picks a pack of candy. What is the probability that the pack chosen is Gummi Bears?

**Problem 2.** Urn  $A$  has 5 white and 7 black balls. Urn  $B$  has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn  $A$  is selected, whereas if the outcome is tails, then a ball from urn  $B$  is selected. Suppose that a white ball is selected. Given this, what is the probability that the coin landed tails?

**Problem 3.** Alice and Bob hid a present for their grandmother. With probability 0.6, the present was hidden by Alice; with probability 0.4, it was hidden by Bob. When Alice hides a present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Bob is equally likely to hide it upstairs or downstairs.

- a. What is the probability that the present is upstairs?
- b. Given that it is downstairs, what is the probability it was hidden by Alice?

**Problem 4.** A lie-detector test, also called a polygraph, is often given when hiring employees for sensitive positions, but some studies have shown there are issues with their use. According to a 1987 study,

- there is an 88% chance of a positive reading (meaning the test says the subject is lying) when the subject is lying,
- there is an 86% chance of a negative reading (meaning the test says the subject is not lying) when the subject is not lying.

Suppose that on a certain question, there is a 99% chance that the subject is not lying. If the test gives a positive reading, what is the conditional probability that the test is incorrect and the subject is not lying?

**Problem 5** (Bonus challenge problem). A study of automobile accidents produced the data in the table below. Suppose an automobile from one of the model years 2012, 2013, or 2014 was involved in an accident. Find the probability that it was from model year 2014.

Model year	Proportion of all vehicles	Proportion involved in an accident from that model year
2014	0.16	0.05
2013	0.18	0.02
2012	0.20	0.03
other	0.46	0.04