

Math 342 — Law of total probability and Bayes' formula

Problem 1. Amy has two bags of candy. The first bag contains two packs of M&Ms and three packs of Gummi Bears. The second bag contains four packs of M&Ms and two packs of Gummi Bears. Amy chooses a bag at random—the first chosen with probability $1/4$ and the second bag chosen with probability $3/4$ — and then picks a pack of candy. What is the probability that the pack chosen is Gummi Bears?

Problem 2. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. Given this, what is the probability that the coin landed tails?

Problem 3. Alice and Bob hid a present for their grandmother. With probability 0.6, the present was hidden by Alice; with probability 0.4, it was hidden by Bob. When Alice hides a present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Bob is equally likely to hide it upstairs or downstairs.

- a. What is the probability that the present is upstairs?
- b. Given that it is downstairs, what is the probability it was hidden by Alice?

Problem 4. A lie-detector test, also called a polygraph, is often given when hiring employees for sensitive positions, but some studies have shown there are issues with their use. According to a 1987 study,

- there is an 88% chance of a positive reading (meaning the test says the subject is lying) when the subject is lying,
- there is an 86% chance of a negative reading (meaning the test says the subject is not lying) when the subject is not lying.

Suppose that on a certain question, there is a 99% chance that the subject is not lying. If the test gives a positive reading, what is the conditional probability that the test is incorrect and the subject is not lying?