

Math 342 — Independence, random variables

Problem 1. A gambler's dispute in 1654 is to have led to the creation of the European school of mathematical probability. Two French mathematicians, Pascal and Fermat, considered the probability that, in 24 throws of a pair of dice, at least one "double six" occurs. It was commonly believed by gamblers at the time that betting on double sixes in 24 throws of a pair of dice would be a profitable bet (ie. greater than 50% probability), but Pascal and Fermat showed otherwise. Find the probability.

Problem 2. A manufacturing process produces electronic components that are sometimes defective. There is a one-in-a-thousand chance that an individual component is defective, and whether or not a component is defective is independent of any other component's status. Find the probability that among 500 components, at least one is defective.

Problem 3. Suppose X is a random variable that takes values on all positive integers. That is, its range is all positive integers. Let $A = \{2 \leq X \leq 4\}$ and $B = \{X \geq 4\}$. Describe the events $A^c, B^c, AB, A \cup B$.

Problem 4. In Julia's garden, there is a 3% chance that a tomato will be bad, with each tomato independent from the others. Julia harvests 100 tomatoes. Let X be the number of bad tomatoes harvested.

- Find the range of X .
- Express the event of getting no bad tomatoes in terms of X and find its probability.
- Express the event of getting at most two bad tomatoes in terms of X and find its probability.