

# Math 102, Fall 2022 — Exam 1 revisions

Mount Holyoke College

Due October 24 at 5:00 pm

**Instructions.** As part of Exam 1, we will have a process for exam corrections. This will give you an opportunity to learn from mistakes and continue to understand the material more deeply. It will also allow you to earn back some credit on missed problems. For each problem that you lost points on, I will ask you to:

- a. Redo the problem and give a correct solution.
- b. Write a short summary of what went wrong in your original solution and how you addressed the issue. This is open ended and it doesn't have to be a long summary (a few sentences is fine), but it should be reflective and written in complete sentences.
- c. Do the corresponding problem from the list below.

The corrections are optional, but will give you a chance to earn back up to 25% of points missed. You can work with others and get help from me and TA's, as well as use the book and class notes. They will be due on Gradescope on October 24 at 5:00 pm. If your score was below 35 points originally, I'll ask you to come to office hours or make a short appointment with me so that we can look through your exam and talk through what can be done to improve your experience in the class overall.

**Problem 1** (2 points). For each limit below, either state that the limit does not exist or give the value of the limit. No justification is needed.

a.  $\lim_{n \rightarrow \infty} \frac{2n^3 + 5n + 7}{6n^5 + 3n^2 + 1}$

b.  $\lim_{n \rightarrow \infty} (-1)^n \frac{3n^6 + 5n + 7}{7n^6 + 3n^2 + 1}$

**Problem 2** (4 points). Find the sum of the following infinite series or explain why it diverges:

$$7 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \dots$$

**Problem 3** (5 points). A patient takes a 200 mg tablet of a drug at 8:00 am each morning. By the end of the 24 hour period, just before taking their daily tablet, 80% of the drug has been absorbed into their blood stream.

- How much of the drug is in their body the moment before taking their 20th tablet?
- How much of the drug is in their body in the long run (ie. in the moment before taking their  $n$ th tablet as  $n \rightarrow \infty$ )?

**Problem 4** (15 points). Determine whether each of the the following series converges or diverges. Make sure to provide justifying computations with clear notation and algebraic work, and at the bottom of each page circle your conclusion and state which tests were used.

a.  $\sum_{n=1}^{\infty} (-1)^n \frac{n^{1/3} + n^{1/4}}{n^2 + n + 1}$

b.  $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{4^n}$

c.  $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{3n^2 + 4n + 5}}{2n + 1}$

**Problem 5** (5 points). For the following series, decide whether it converges or diverges, and then prove your claim using the Comparison Test:

$$\sum_{n=1}^{\infty} \frac{2^n - n}{3^n + 4n}.$$

**Problem 6** (10 points). State whether each of the following series converges absolutely, converges conditionally, or diverges. Give a brief explanation of what test(s) should be used to give a rigorous justification, but please skip any detailed calculations.

a.  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+5}}$

b.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n^2 + 3}$

**Problem 7** (9 points). Answer the following; no justification needed unless otherwise noted.

- Give an example where  $\lim_{n \rightarrow \infty} a_n = 0$  and  $\sum_{n=1}^{\infty} 2 + a_n$  converges or explain in a sentence why no such example is possible.
- Is it true that if  $\sum_{n=1}^{\infty} a_n$  converges, then  $\sum_{n=1}^{\infty} \sqrt{a_n}$  must converge too? If you believe it's true, explain why. If you believe it's not true, give an example where the statement fails.
- Give an example of a series  $\sum_{n=1}^{\infty} a_n$  whose partial sum sequence diverges.