- Tim / Prof. / Prof. Chumley (he/him)
- Moodle - announcements
- Webpage (tchumley.mtholyoke.edu/m102)
  - notes, worksheets, homework, syllabus
  - updated daily
- Homework (weekly)
  - written problems, submitted on Gradescope
  - due Fridays at 5 pm
- Quizzes - Wednesdays (first one is Sep 14)
- Exams - two during semester, one during finals
- Participation - come to class, be a good community member, stay in touch when something goes wrong (eg. illness)
- Office hours (tentative)
  - Tuesdays 9:15-10:15
  - Wednesday 4-5
  - Thursdays 4:30-5:30

  drop in (Clapp 423), no appointment necessary

Overview of Math 102

1. sequences and series
2. integration techniques
3. Taylor series
4. applications (probability, differential eq's)

Opening question:

How does a computer/calculator compute values like $\ln(2)$, $\sin(3)$, or e^4?

We'll soon learn these functions can be represented as infinite sums of powers of x:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

First, we need to learn some basics...

§ 9.1 Sequences

Definition A sequence is an infinite list of numbers in a specified order:

$s_1, s_2, s_3, \ldots$

Notation s_n represents the n^{th} term

We use braces $\{s_1, s_2, \ldots\}$ or $\{s_n\}$ when talking about all the terms

Examples

① Fibonacci sequence

② $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ $s_n = \frac{1}{n}$

(called the harmonic sequence)

③ $2, 4, 8, 16, 32, 64, \ldots$ $s_n = 2^n$

④ Amount of antibiotic in your body n hours after a dose.

Question Can you write a general formula for s_n in examples 2, 3 above?

Definition If the terms of a sequence $\{s_n\}$ approach a finite value L as $n \to \infty$, we say the sequence **converges** and we call L the **limit** of the sequence.

We often use the notation $\lim_{n \to \infty} s_n$ when discussing limits.

If the terms don't approach any finite value as $n \to \infty$, we say $\{s_n\}$ **diverges**.

Question Do any of the sequences in our last examples converge?

(2) converges to 0

(1) and (3) diverge (and we say their limit is $+\infty$)

(4) we'll talk about next time

__Examples__ Write the general formula for S_n and explain whether the following sequences converge.

① $1, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \frac{-1}{6}, \ldots$

$S_n = \frac{(-1)^{n+1}}{n}$, $\lim_{n \to \infty} \frac{(-1)^{n+1}}{n} = 0$ (converges to 0)

② $-1, 3, -9, 27, -81, 216, \ldots$

$S_n = (-1)^n 3^{n-1}$ $\lim_{n \to \infty} (-1)^n 3^{n-1}$ does not exist (diverges)

③ $\frac{5}{2}, \frac{-5}{4}, \frac{5}{6}, \frac{-5}{8}, \frac{5}{10}, \ldots$

$S_n = \frac{(-1)^{n+1} 5}{2n}$

__Lessons__ ① use $(-1)^n$ or $(-1)^{n+1}$ to get alternating signs

② sequences with alternating signs can converge (and some diverge)

Examples Find $\lim_{n\to\infty} s_n$ if the sequence converges or explain why it diverges.

① $$\lim_{n\to\infty} \frac{3n^2 - n + 3}{5n^2 + 7} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{3 - \frac{1}{n} + \frac{3}{n^2}}{5 + \frac{7}{n^2}}$$

$$= \frac{3 - 0 + 0}{5 + 0} = \frac{3}{5}$$

(converges to $\frac{3}{5}$)

② $$\lim_{n\to\infty} \frac{10n^2 + n + 4}{3n^5 + n - 8} \cdot \frac{\frac{1}{n^5}}{\frac{1}{n^5}}$$

$$= \lim_{n\to\infty} \frac{\frac{10}{n^3} + \frac{1}{n^4} + \frac{4}{n^5}}{3 + \frac{1}{n^4} - \frac{8}{n^5}} = \frac{0 + 0 + 0}{3 + 0 + 0} = 0$$

(converges to 0)

③ $$\lim_{n\to\infty} \frac{4n^6 + n^3 + 5}{3n^5 + n^2 + 4n} \cdot \frac{\frac{1}{n^6}}{\frac{1}{n^6}}$$

$$= \lim_{n\to\infty} \frac{4 + \frac{1}{n^3} + \frac{5}{n^6}}{\frac{3}{n} + \frac{1}{n^4} + \frac{4}{n^5}} = \frac{4 + 0 + 0}{0 + 0 + 0} = +\infty$$

(diverges to $+\infty$)

Lesson

- The limit is ratio of leading coefficients when leading powers are equal
- The limit is 0 when leading power on top is smaller than leading power on bottom
- The limit is $+\infty$ when leading power on top is larger.