- Tim / Prof. / Prof. Chumley (he/him)
- Moodle – announcements, Q+A
- Webpage (tchumley.mtholyoke.edu/m102)
  - notes, worksheets, homework, syllabus
  - updated daily
- Homework (weekly)
  - submitted on Gradescope
  - due Fridays at 5 pm
  - redos (details to be announced)
- Quizzes – most Fridays, 15 minutes at end of class
- Exams – two midterms, one final (in-class)
- Participation – come to class, ask and answer questions, be a good community member, stay in touch when something goes wrong (eg. illness)
- Office hours (tentative)
  - Mondays 4:00-5:00
  - Wednesdays 4:00-5:00
  - Thursdays 1:00-2:00

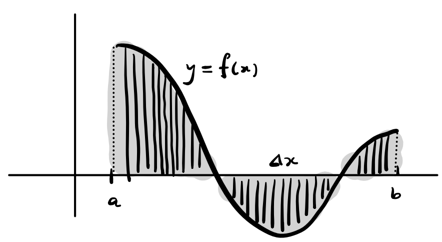
  drop in (Clapp 423), no appointment necessary

Chapter 5 and 6 Review

Let $f(x)$ be a continuous function. The <u>definite integral</u> of f over the interval $[a, b]$, denoted

$$\int_a^b f(x)\,dx$$

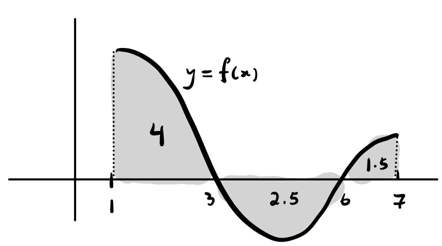
is the <u>signed</u> area between f and the x-axis for values of x in $[a, b]$. The precise definition is given in terms of limits and Riemann sums of rectangular areas.



$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_i \underbrace{f(x_i)}_{\text{height}} \underbrace{\Delta x}_{\text{width}}$$

We'll keep this precise definition in mind but only go into these details as needed.

<u>Example</u>



numbers in shaded regions are <u>unsigned</u> areas

$$\int_1^7 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^7 f(x)\,dx$$

$$= 4 + (-2.5) + 1.5$$

$$= 3$$

Properties of Definite Integrals

① $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

② $\displaystyle\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

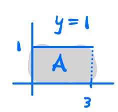
③ $\displaystyle\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

④ $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

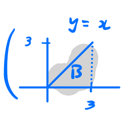
Example If possible, use geometry to compute the following:

① $\displaystyle\int_0^3 (1+2x)\,dx$

$\displaystyle = \int_0^3 1\,dx + 2\int_0^3 x\,dx =$



$+ \; 2 \Big($

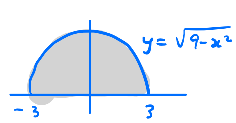


$\Big)$

$= \text{area}(A) + 2\,\text{area}(B)$

$= 3 + 2\left(\frac{1}{2}(3)^2\right) = 12$

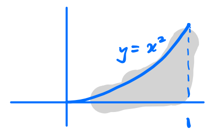
② $\displaystyle\int_{-3}^3 \sqrt{9-x^2}\,dx$



$=$ area of half-disk

$= \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi.$

③ $\int_0^1 x^2\,dx$



This area is not so easy with just basic geometry!

Definition Let $f(x)$ be a given function. Then $F(x)$ is called an antiderivative of $f(x)$ if $F'(x) = f(x)$.

We use the term indefinite integral, denoted $\int f(x)\,dx$, for a generic antiderivative of $f(x)$.

Example Find

① $\int x\,dx = \frac{1}{2}x^2 + C$

② $\int x^2\,dx = \frac{1}{3}x^3 + C$

③ $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = -\frac{1}{2}x^{-2} + C$

④ $\int \frac{1}{x^4}\,dx = \int x^{-4}\,dx = -\frac{1}{3}x^{-3} + C$

⑤ $\int \frac{1}{x}\,dx = \ln|x| + C$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ when $n \neq -1$.

Theorem (Fundamental Theorem of Calculus, part I)

Let f be a continuous function on $[a,b]$ and let F be an antiderivative of f. Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$= F(x)\Big|_a^b$$

Example Compute the following using the FTOC.

① $\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}(1^3 - 0^3) = \frac{1}{3}$

② $\int_1^2 (4x^3+4x+1)\,dx = x^4 + 2x^2 + x\Big|_1^2$

$= 2^4 + 2^3 + 2 - (1+2+1) = 26 - 4 = 22$

③ $\int_0^1 3\sqrt{x}\,dx = 3\int_0^1 x^{1/2}\,dx = 3\left(\frac{2}{3}x^{3/2}\Big|_0^1\right) = 2$

④ $\int_0^5 e^x\,dx = e^x\Big|_0^5 = e^5 - 1$

⑤ $\int_{-\pi}^{\pi} \sin x\,dx = -\cos x\Big|_{-\pi}^{\pi} = \cos x\Big|_{\pi}^{-\pi} = -1-(-1) = 0.$

Antiderivatives to remember

- $\int k\,dx = kx + C$ when k is a given constant
- $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ when $n \neq -1$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \sin x\,dx = -\cos x + C$