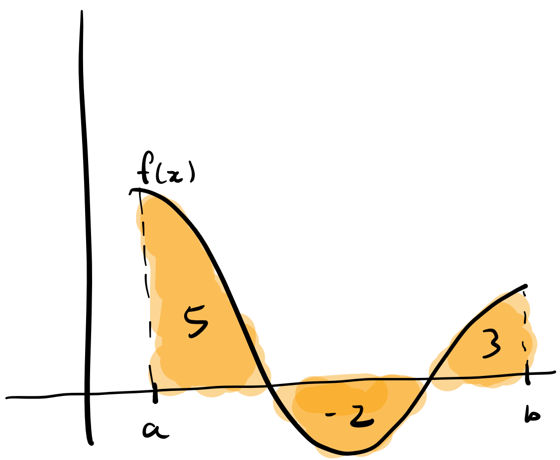
Review of integral basics (5.1, 5.2, 5.4)

The definite integral of a function f over the interval from $x = a$ to $x = b$

is $$\int_a^b f(x)\,dx = \text{the signed area between } f \text{ and the } x\text{-axis over } [a, b].$$

Example

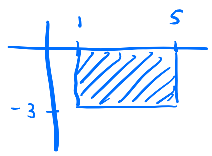


$$\int_a^b f(x)\,dx = 5 - 2 + 3 = 6$$

It's defined using Riemann sums, a concept you probably saw in calculus 1, but we'll only go into those details as needed

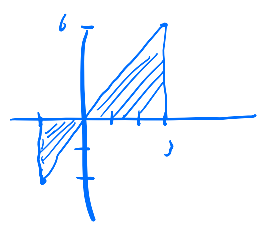
Example Find the following integrals using the graphs of functions and geometry:

① $\displaystyle\int_1^5 -3\,dx = 4(-3) = -12$



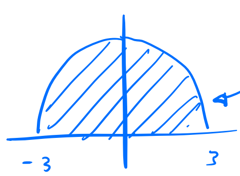
② $\displaystyle\int_{-1}^3 2x\,dx = -\frac{1}{2}(1)(2) + \frac{1}{2}(3)(6)$

$= -1 + 9 = 8$



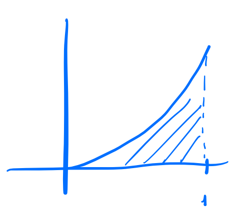
③ $\displaystyle\int_{-3}^3 \sqrt{9-x^2}\,dx$

$= \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$



Semicircle $\sqrt{9-x^2}$

④ $\displaystyle\int_0^1 x^2\,dx$



tough to do geometry here...

We won't always be able to use geometry...

Fundamental theorem of calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Note sometimes we see the theorem written as:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$= F(x)\Big|_a^b$$

where $F'(x) = f(x)$

We call $F(x)$ an antiderivative of $f(x)$

when $F'(x) = f(x)$.

Antiderivatives to know:

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \text{ when } n \neq -1$$

$$\int x^{-1}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int e^x\,dx = e^x + C$$

See Theorem 5.1.2 in textbook for full list of antiderivatives to know.

Examples Find the following integrals using the FTOC

① $\displaystyle\int_1^3 (3x^2 + 4x + 1)\,dx$

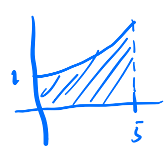
$$= x^3 + 2x^2 + x \Big|_1^3 = (27 + 18 + 3) - (1 + 2 + 1)$$

$$= 48 - 4 = 42$$

② $\displaystyle\int_0^1 (5x^3 - 3\sqrt{x})\,dx$

$$= \frac{5}{4}x^4 - 2x^{3/2} \Big|_0^1 = \frac{5}{4} - 2 = -\frac{3}{4}$$

③ $\displaystyle\int_0^5 e^x\,dx = e^x \Big|_0^5 = e^5 - 1$



④ $\displaystyle\int_0^{\pi} \sin x\,dx = -\cos x \Big|_0^{\pi} = -(\cos\pi - \cos 0)$

$$= 2$$