More integration review

Properties

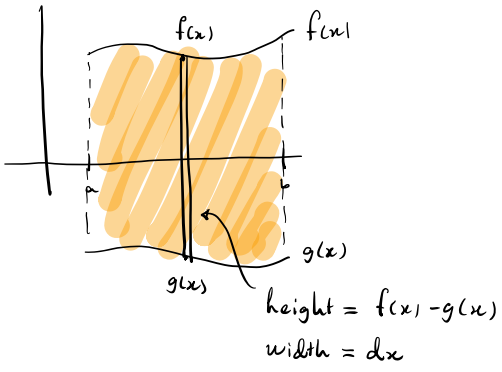
$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

- areas between curves



Area between top curve $f(x)$ and bottom curve $g(x)$ between $x=a$ and $x=b$ is

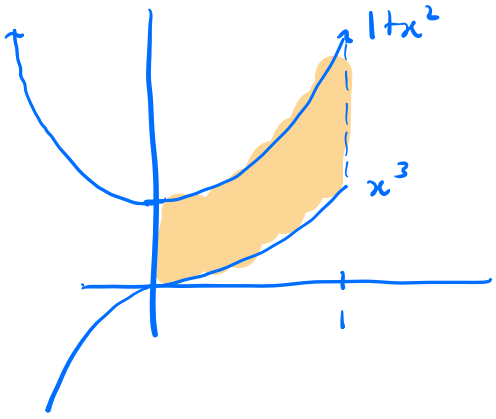
$$\int_a^b [f(x) - g(x)]\,dx.$$

Note this is an unsigned area (always positive when $f(x)$ is top curve and $g(x)$ is bottom curve)

Example Find the area between

$y = 1 + x^2$ and $y = x^3$

between $x = 0$ and $x = 1$.



$$\int_0^1 \left[(1+x^2) - (x^3)\right] dx$$

$$= \int_0^1 \left(1 + x^2 - x^3\right) dx$$

$$= x + \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1$$

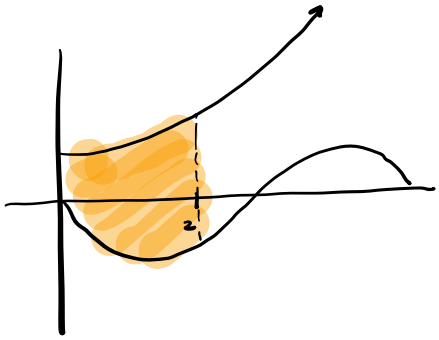
$$= 1 + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{13}{12}$$

Example Find the area between

$y = -\sin x$ and $y = e^x$

for x-values between 0 and 2.



$$\int_0^2 \left[\underbrace{e^x}_{\text{top}} - \underbrace{(-\sin x)}_{\text{bottom}} \right] dx$$

$$= \int_0^2 (e^x + \sin x)\, dx$$

$$= e^x - \cos x \Big|_0^2$$

$$= \left[e^2 - \cos(2)\right] - \left[e^0 - \cos(0)\right]$$

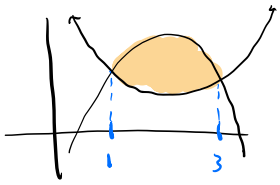
$$= e^2 - \cos 2$$

Example Find the area of the bounded region between

$y = x^2 - 4x + 5$ (bottom)

and $y = -x^2 + 4x - 1$ (top)

shown below



Where do these curves intersect?

$$x^2 - 4x + 5 = -x^2 + 4x - 1$$

$$2x^2 - 8x + 6 = 0$$

$$2(x^2 - 4x + 3) = 0$$

$$2(x-3)(x-1) = 0$$

$$x = 1, 3$$

$$\text{Area} = \int_1^3 \left[(-x^2 + 4x - 1) - (x^2 - 4x + 5)\right] dx$$

$$= \int_1^3 \left(-2x^2 + 8x - 6\right) dx$$

$$= -\frac{2}{3}x^3 + 4x^2 - 6x \Big|_1^3$$

$$= \left[-\frac{2}{3}(27) + 4(9) - 6(3)\right] - \left[-\frac{2}{3} + 4 - 6\right]$$

$$= \left[-18 + 36 - 18\right] - \left[-\frac{2}{3} - 2\right]$$

$$= \frac{8}{3}.$$