Calc I review

How do we compute the derivative of $f(g(x))$?

we use $$\frac{d}{dx}\Big(f(g(x))\Big) = f'(g(x))\, g'(x)$$

(the chain rule)

Example Find the following derivatives:

① $$\frac{d}{dx}\Big(\cos(x^3)\Big) = \sin(x^3)\cdot 3x^2$$

② $$\frac{d}{dx}\Big(\sin(x^3)\Big) = \cos(x^3)\cdot 3x^2$$

③ $$\frac{d}{d\theta}\Big(e^{\cos\theta}\Big) = e^{\cos\theta}\cdot(-\sin\theta) = -\sin\theta\, e^{\cos\theta}$$

6.1 Method of Substitution

Our goal today is to learn a method of finding integrals that's like the chain rule in reverse.

Goal Compute integrals of the form

$$\int f(\underbrace{g(x)}_{u})\,\underbrace{g'(x)\,dx}_{du} = F(g(x)) + C$$

where $F(x)$ is an antiderivative of $f(x)$

We'll call this method u-substitution or just substitution.

Examples

① $$\int x \sin(x^2+5)\,dx$$

$u = x^2+5$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + C = \frac{1}{2}\cos(x^2+5) + C$$

② $$\int \cos(5x)\,dx$$

$u = 5x$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$$= \frac{1}{5}\int \cos u\,du$$

$$= \frac{1}{5}\sin u + C = \frac{1}{5}\sin(5x) + C$$

③ $$\int \frac{7}{-3x+1}\,dx$$

$u = -3x+1$

$du = -3\,dx$

$-\frac{1}{3}du = dx$

$$= -\frac{7}{3}\int \frac{1}{u}\,du$$

$$= -\frac{7}{3}\ln|u| + C = -\frac{7}{3}\ln|-3x+1| + C$$

④ $\int \sin x \cos x \, dx$ $\qquad u = \sin x$

$du = \cos x \, dx$

$= \int u \, du$

$= \frac{1}{2} u^2 + C = \frac{1}{2} \sin^2 x + C$

⑤ $\int \frac{1}{x \ln x} dx$ $\qquad u = \ln x$

$du = \frac{1}{x} dx$

$= \int \frac{1}{x} \cdot \frac{1}{\ln x} dx$

$= \int \frac{1}{u} du = \ln|u| + C = \ln|\ln x| + C$

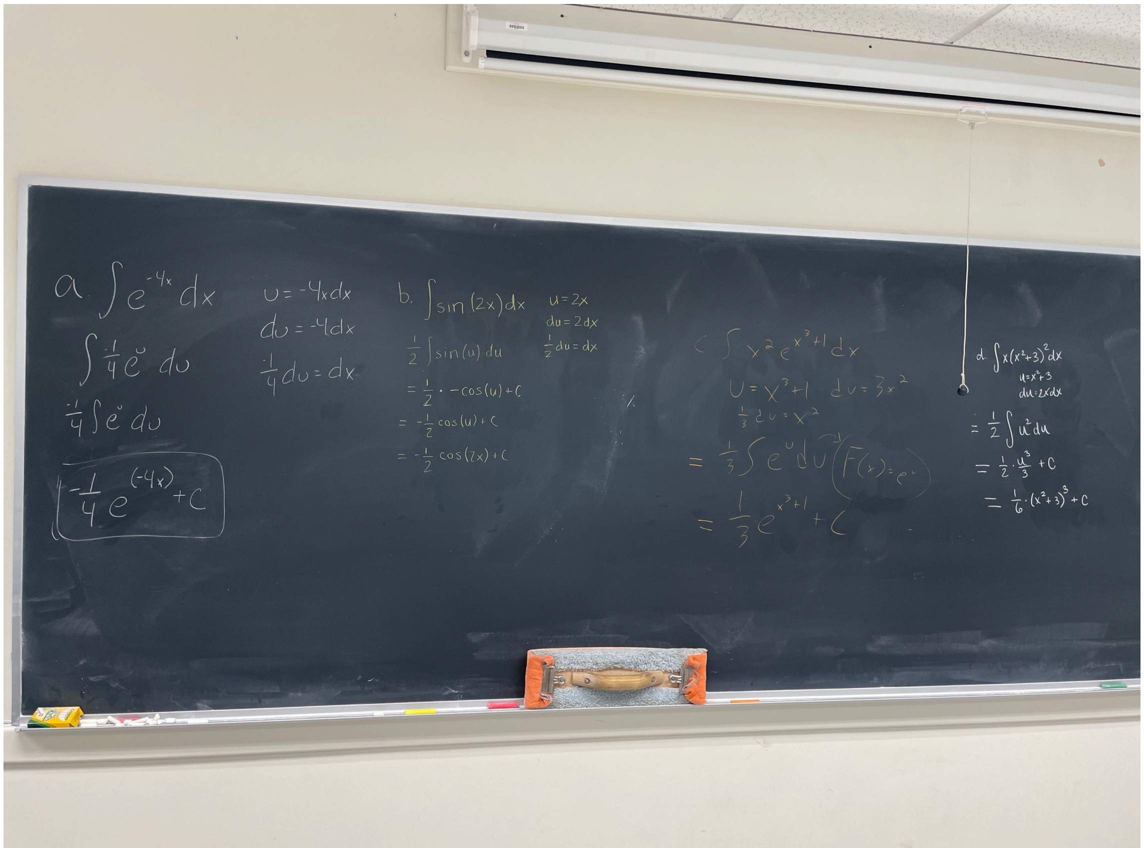
⑥ $\int x \sqrt{x+3} \, dx$ $\qquad u = x+3, \quad x = u-3$

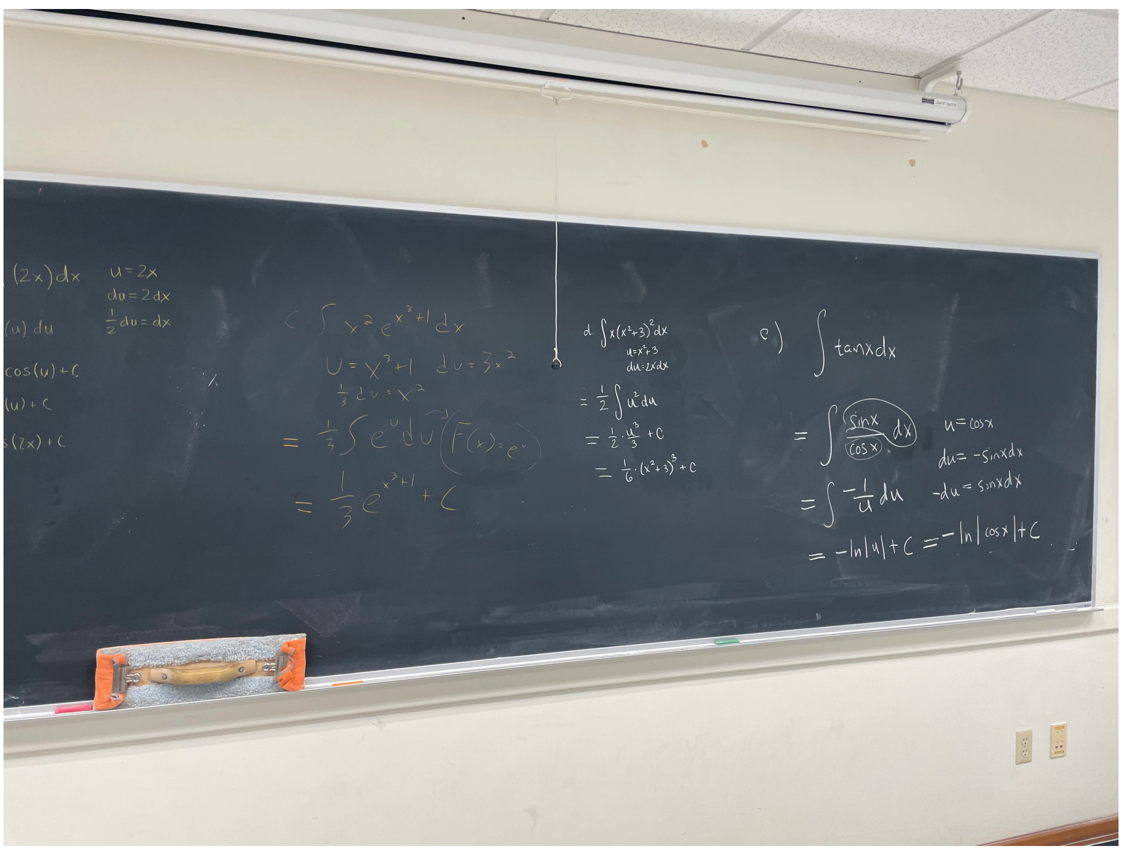
$du = dx$

$= \int (u-3)\sqrt{u} \, du$

$= \int (u^{3/2} - 3u^{1/2}) du = \frac{2}{5} u^{5/2} - 2u^{3/2} + C$

$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C.$


a $\int e^{-4x}\,dx$
$u=-4x\,dx$
$du=-4dx$
$-\frac{1}{4}du=dx$
$\int -\frac{1}{4}e^{u}\,du$
$-\frac{1}{4}\int e^{u}\,du$
$\boxed{-\frac{1}{4}e^{(-4x)}+C}$
b. $\int \sin(2x)\,dx$
$u=2x$
$du=2dx$
$\frac{1}{2}du=dx$
$\frac{1}{2}\int \sin(u)\,du$
$=\frac{1}{2}\cdot -\cos(u)+C$
$=-\frac{1}{2}\cos(u)+C$
$=-\frac{1}{2}\cos(2x)+C$
c $\int x^2e^{x^3+1}\,dx$
$U=x^3+1 \quad du=3x^2$
$\frac{1}{3}du=x^2$
$=\frac{1}{3}\int e^{u}\,du$
$F(x)=e^{x}$
$=\frac{1}{3}e^{x^3+1}+C$
d $\int x(x^2+3)^2\,dx$
$u=x^2+3$
$du=2xdx$
$=\frac{1}{2}\int u^2\,du$
$=\frac{1}{2}\cdot\frac{u^3}{3}+C$
$=\frac{1}{6}\cdot(x^2+3)^3+C$


e) $\int \tan x\,dx$
$=\int \frac{\sin x}{\cos x}\,dx$
$u=\cos x$
$du=-\sin x\,dx$
$-du=\sin x\,dx$
$=\int -\frac{1}{u}\,du$
$=-\ln|u|+C=-\ln|\cos x|+C$

(g) $\int \frac{\sin(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int \sin(u)\,du$

$= -\cos(u) + C$

$= -\cos(\ln x) + C.$

(h) $\int \frac{\cos(5x)}{e^{\sin(5x)}}\,dx = \int \frac{1/5}{e^u}\,du = \frac{1}{5}\int \frac{1}{e^u}\,du$

$u = \sin(5x)$

$du = 5\cos(5x)\,dx$

$\frac{1}{5}\,du = \cos(5x)\,dx$

$= \frac{1}{5}\int e^{-u}\,du$

$w = -u$

$dw = -du$

$-dw = du$

$= \frac{-1}{5}\int e^{w}\,dw$

$= \frac{-1}{5}e^{w} + C$

$= \frac{-1}{5}e^{-u} + C = \frac{-1}{5}e^{-\sin(5x)} + C.$