(a) $\int \frac{dx}{5-3x}$ $w = 5-3x$

$dw = -3dx$

$= -\frac{1}{3}\int \frac{1}{w}\,dw$ $-\frac{1}{3}dw = dx$

$= -\frac{1}{3}\ln|w| + C = -\frac{1}{3}\ln|5-3x| + C$

(b) $\int x\cos(5x)\,dx$ $u = x$ $dv = \cos(5x)dx$

$du = dx$ $v = \frac{1}{5}\sin(5x)$

$= \frac{1}{5}x\sin(5x) - \frac{1}{5}\int \sin(5x)dx$

$= \frac{1}{5}x\sin(5x) + \frac{1}{25}\cos(5x) + C$

(c) $\int \frac{(\ln x)^2}{x}dx$ $w = \ln x$

$dw = \frac{1}{x}dx$

$= \int w^2\,dw$

$= \frac{1}{3}w^3 + C = \frac{1}{3}\ln^3 x + C$

(d) $\int_1^e x^3 \ln x\,dx$ $u = \ln x$ $dv = x^3 dx$

$du = \frac{1}{x}dx$ $v = \frac{1}{4}x^4$

$= \frac{1}{4}x^4\ln x\Big|_1^e - \int_1^e \frac{1}{4}x^3dx$

$= \frac{1}{4}e^4 - \frac{1}{16}x^4\Big|_1^e$

$= \frac{1}{4}e^4 + \frac{1}{16} - \frac{1}{16}e^4$

$= \frac{3}{16}e^4 + \frac{1}{16}$

(e) $\int x(2x^2+5)^8\,dx$ $\quad w = 2x^2+5$

$dw = 4x\,dx$

$= \frac{1}{4}\int w^8\,dw$ $\quad \frac{1}{4}dw = x\,dx$

$= \frac{1}{36} w^9 + C = \frac{1}{36}(2x^2+5)^9 + C$

(f) $\int (x^2+2x)\cos x\,dx$ $\quad u = x^2+2x \quad dv = \cos x\,dx$

$du = (2x+2)dx \quad v = \sin x$

$= (x^2+2x)\sin x - \int (2x+2)\sin x\,dx$

$u = 2x+2 \quad dv = \sin x\,dx$

$du = 2dx \quad v = -\cos x$

$= (x^2+2x)\sin x - \left(-(2x+2)\cos x + \int 2\cos x\,dx\right)$

$= (x^2+2x)\sin x + (2x+2)\cos x - 2\sin x + C$

(g) $\int \sqrt{x}\sin(1+x^{3/2})\,dx$ $\quad w = 1+x^{3/2}$

$dw = \frac{3}{2}x^{1/2}\,dx$

$= \frac{2}{3}\int \sin w\,dw$ $\quad \frac{2}{3}dw = x^{1/2}dx$

$= -\frac{2}{3}\cos w + C = -\frac{2}{3}\cos(1+x^{3/2}) + C.$

(h)

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx \qquad w = \frac{1}{x}$$

$$dw = -\frac{1}{x^2}\,dx$$

$$= -\int_1^{1/2} e^w\,dw \qquad -dw = \frac{1}{x^2}\,dx$$

$$= \int_{1/2}^{1} e^w\,dw = e^w \Big|_{1/2}^{1} = e - e^{1/2}$$

(i)

$$\int \frac{3x^2+8x}{x^3+4x^2+1}\,dx \qquad w = x^3+4x^2+1$$

$$dw = (3x^2+8x)\,dx$$

$$= \int \frac{1}{w}\,dw$$

$$= \ln|w| + C = \ln\left|x^3+4x^2+1\right| + C$$

(j)

$$\int \frac{x^2+x+1}{x^3+x^2-2x}\,dx$$

$$\frac{x^2+x+1}{x(x^2+x-2)} = \frac{x^2+x+1}{x(x+2)(x-1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$x^2+x+1 = A(x+2)(x-1) + Bx(x-1) + Cx(x+2)$$

$\underline{x=-2}$ $\quad 3 = 6B \Rightarrow B = 1/2$

$\underline{x=0}$ $\quad 1 = -2A \Rightarrow A = -1/2$

$\underline{x=1}$ $\quad 3 = 3C \Rightarrow C = 1$

$$\int \frac{-1/2}{x}\,dx + \int \frac{1/2}{x+2}\,dx + \int \frac{1}{x-1}\,dx$$

$$= -\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x+2| + \ln|x-1| + c$$

(b) $$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$10 = A(x^2+9) + (Bx+C)(x-1)$$

$\underline{x=1}$ $\quad 10 = 10A, \quad A=1$

$\underline{x=0}$ $\quad 10 = 9 - C, \quad C = -1$

$\underline{x=2}$ $\quad 10 = 13 + 2B - 1, \quad B = -1$

$$\int \frac{1}{x-1}\,dx + \int \frac{-x-1}{x^2+9}\,dx$$

$$= \ln|x-1| + \int \frac{-x}{x^2+9}\,dx + \int \frac{-1}{x^2+9}\,dx$$

$u = x^2+9$ $\qquad x = 3\tan\theta$

$du = 2x\,dx$ $\qquad dx = 3\sec^2\theta\,d\theta$

$\frac{1}{2}du = x\,dx$

$$= \ln|x-1| - \frac{1}{2}\int \frac{1}{u}\,du - \int \frac{1}{9\tan^2\theta+9}\,3\sec^2\theta\,d\theta$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\int \frac{1}{\tan^2\theta+1}\sec^2\theta\,d\theta$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\int d\theta$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\theta + C$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

c) $\displaystyle\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

$$\frac{x^2-5x+16}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2-5x+16 = A(x-2)^2 + B(2x+1)(x-2) + C(2x+1)$$

$\underline{x=-\frac{1}{2}}$ $\quad \frac{1}{4} + \frac{5}{2} + 16 = \frac{25A}{4} \Rightarrow A = 3$

$\underline{x=2}$ $\quad 10 = 5C \Rightarrow C = 2$

$\underline{x=0}$ $\quad 16 = 4A - 2B + C$

$= 12 - 2B + 2$

$B = -1$

$$\int \frac{3}{2x+1}\,dx + \int \frac{-1}{x-2}\,dx + \int \frac{2}{(x-2)^2}\,dx$$

$u = 2x+1$

$du = 2dx$

$$= \frac{3}{2}\int \frac{1}{u}\,du - \ln|x-2| - 2(x-2)^{-1} + C$$

$$= \frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2} + C$$

Problem 4. Evaluate the following improper integrals or show that they diverge.

a. $\int_3^\infty \frac{1}{2(x+5)^2}\,dx$

b. $\int_{-1}^4 \frac{1}{(x+1)^{1/3}}\,dx$

c. $\int_{-3}^3 \frac{1}{x^{2/3}}\,dx$

d. $\int_0^\infty \frac{1}{(x-1)^4}\,dx$

a)

$$\int_3^\infty \frac{1}{2(x+5)^2}\,dx$$

$$= \lim_{b\to\infty} \int_3^b \frac{1}{2(x+5)^2}\,dx$$

$$= \lim_{b\to\infty} \frac{-1}{2}(x+5)^{-1}\Big|_3^b$$

$$= \lim_{b\to\infty} \left(\frac{1}{2}(8)^{-1} - \frac{1}{2}\frac{1}{b+5}\right)$$

$$= \frac{1}{16}$$

b)

$$\int_{-1}^4 \frac{1}{(x+1)^{1/3}}\,dx$$

$$= \lim_{a\to -1^+} \int_a^4 \frac{1}{(x+1)^{1/3}}\,dx$$

$$= \lim_{a\to -1^+} \frac{3}{2}(x+1)^{2/3}\Big|_a^4$$

$$= \lim_{a\to -1^+} \frac{3}{2}(5)^{2/3} - \frac{3}{2}(a+1)^{2/3}$$

$$= \frac{3}{2}(5)^{2/3}$$

c)

$$\int_{-3}^3 \frac{1}{x^{2/3}}\,dx = \int_{-3}^0 \frac{1}{x^{2/3}}\,dx + \int_0^3 \frac{1}{x^{2/3}}\,dx$$

$$= \lim_{b\to 0^-} \int_{-3}^b \frac{1}{x^{2/3}}\,dx + \lim_{a\to 0^+} \int_a^3 \frac{1}{x^{2/3}}\,dx$$

$$= \lim_{b\to 0^-} 3x^{1/3}\Big|_{-3}^b + \lim_{a\to 0^+} 3x^{1/3}\Big|_a^3$$

$$= \lim_{b\to 0^-} 3b^{1/3} - 3(-3)^{1/3} + \lim_{a\to 0^+} 3(3)^{1/3} - 3a^{1/3} = 6(3)^{1/3}$$

d) $\displaystyle\int_0^\infty \frac{1}{(x-1)^3}\,dx$

$$= \int_0^1 \frac{1}{(x-1)^3}\,dx + \int_1^2 \frac{1}{(x-1)^3}\,dx + \int_2^\infty \frac{1}{(x-1)^3}\,dx$$

$$\int_0^1 \frac{1}{(x-1)^3}\,dx = \lim_{b\to 1^-} \int_0^b \frac{1}{(x-1)^3}\,dx$$

$$= \lim_{b\to 1^-} -\frac{1}{2}(x-1)^{-2}\Big|_0^b$$

$$= \lim_{b\to 1^-} \frac{1}{2}(-1)^{-2} - \frac{1}{2}(b-1)^{-2}$$

$$= \frac{1}{2} - \frac{1}{2}\lim_{b\to 1^-} \frac{1}{(b-1)^2}$$

$= -\infty$, so the whole integral

diverges since one term diverges.