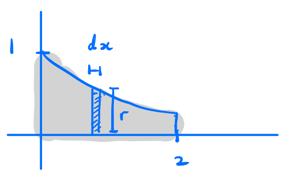
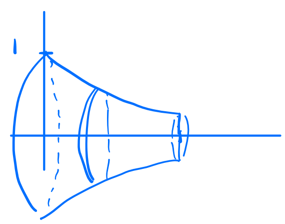
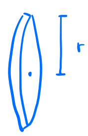
7.2 Volumes of revolution

Example Consider the region bounded by the curves $y = e^{-x}$, $y = 0$, $x = 0$, $x = 2$. Find the volume of the solid formed by revolving this region around the x-axis.



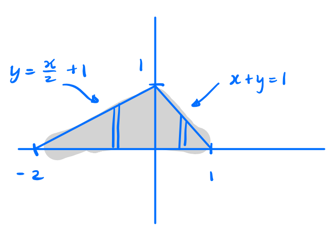




revolving one rectangular slice yields a disk with volume $\pi r^2 \, dx$ and $r = e^{-x}$

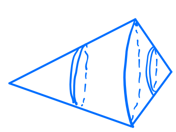
$$\text{Total volume} = \int_0^2 \pi (e^{-x})^2 \, dx = \int_0^2 \pi e^{-2x} \, dx$$

Example Find the volume of the solid formed by revolving the region bounded by $x + y = 1$, $y = \frac{x}{2} + 1$, $y = 0$ around the x-axis.



Between $x = -2$ and $x = 0$, the radius of a disk is $r = \frac{x}{2} + 1$

But between $x = 0$ and $x = 1$, it's $r = 1 - x$

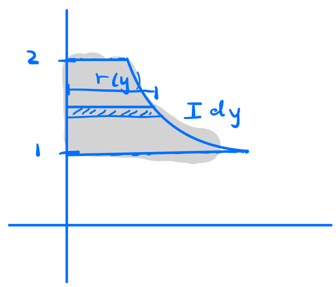


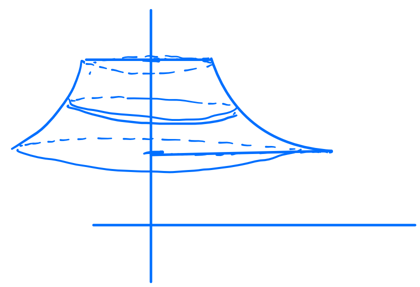
$$\text{total volume} = \int_{-2}^{0} \pi \left(\frac{x}{2} + 1\right)^2 dx + \int_0^1 \pi (1-x)^2 \, dx$$

Example Consider the region bounded by

$y = \frac{1}{\sqrt{x}}$, $y = 1$, $y = 2$, $x = 0$, revolved around

the y-axis. Find the volume.





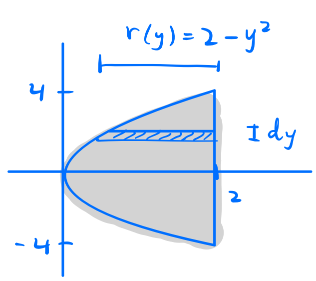
Now the radius of each disk slice is a

function of y: $r(y) = \frac{1}{y^2}$

$$\text{Total volume} = \int_1^2 \pi \left(\frac{1}{y^2}\right)^2 dy$$

Example Consider the region bounded by $x = y^2$, $x = 2$

and revolved around $x = 2$. Find the volume.



$$\int_{-4}^{4} \pi (2 - y^2)^2 dy$$

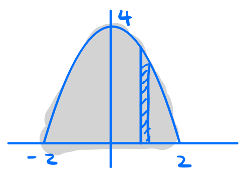
Problem 1. Use the disk method to set up an integral to find the volume of each solid of revolution described below. No need to compute the integrals.

a. Bounded between: $y = 4 - x^2$, $y = 0$. Revolved around: $y = 0$.

b. Bounded between: $y = 3x, x = 1, x = 3$. Revolved around: $y = 0$.

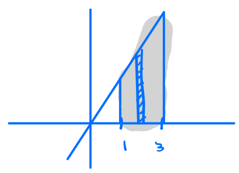
c. Bounded between: $y = \ln x, y = 0, y = 1, x = 0$. Revolved around: $y = 1$.

(a)



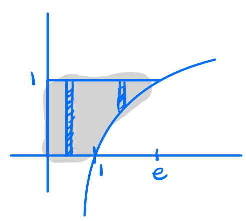
$$\int_{-2}^{2} \pi \left(4 - x^2\right)^2 dx$$

(b)



$$\int_{1}^{3} \pi (3x)^2 dx$$

(c)



$$\int_{0}^{1} \pi (1)^2 dx + \int_{1}^{e} \pi (1 - \ln x)^2 dx$$

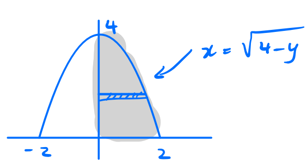
Problem 2. Use the disk method to set up an integral to find the volume of each solid of revolution described below. No need to compute the integrals.

a. Bounded between: $y = 4 - x^2, y = 0, x = 0$. Revolved around: $x = 0$.

b. Bounded between: $y = 3x, y = 0, x = 3$. Revolved around: $x = 3$. addendum: $x=1$ also a bounding curve

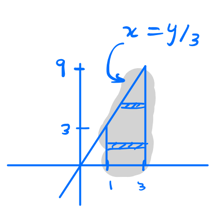
c. Bounded between: $y = x^2, x = 0, x + y = 2$ and in the first quadrant. Revolved around $x = 0$.

(a)



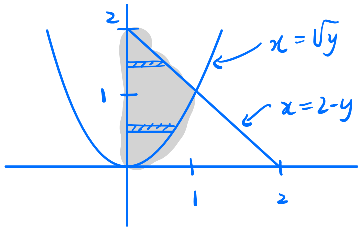
$$\int_{0}^{4} \pi \left(\sqrt{4-y}\right)^2 dy$$

(b)



$$\int_{0}^{3} \pi (2)^2 dy + \int_{3}^{9} \pi (3 - y/3)^2 dy$$

(c)



intersection point:

$$x^2 = 2 - x$$

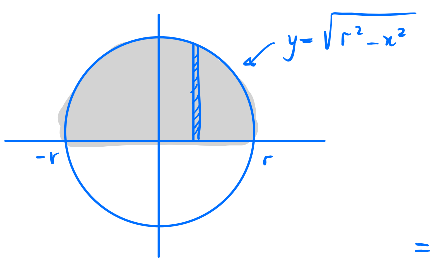
$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1$$

$$\int_0^1 \pi (\sqrt{y})^2 \, dy + \int_1^2 \pi (2-y)^2 \, dy$$

Problem 3. Let $r > 0$ be a given constant. Use the disk method to find the volume of the solid formed by revolving the region bounded by $y = \sqrt{r^2 - x^2}$ and $y = 0$ around the x-axis. Make sure to set up and compute the integral to get a value in terms of r. What formula did you derive?



$$\int_{-r}^{r} \pi \left(\sqrt{r^2 - x^2} \right)^2 dx$$

$$= \int_{-r}^{r} \pi (r^2 - x^2) \, dx$$

$$= 2\pi \int_0^r (r^2 - x^2) \, dx$$

$$= 2\pi \left(r^2 x - \frac{1}{3} x^3 \Big|_0^r \right)$$

$$= 2\pi \left(r^3 - \frac{1}{3} r^3 \right)$$

$$= \frac{4}{3} \pi r^3$$