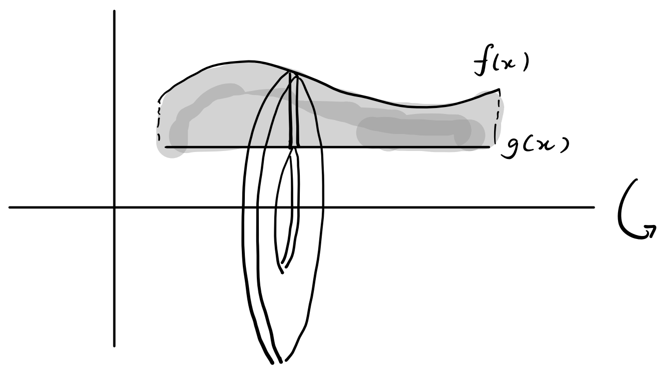
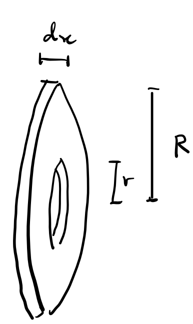
7.2 Washer Method

Sometimes we revolve a region around an axis, but there is a gap between the region and the axis:



In this case our solid has a hole through the middle and each slice is a disk with a hole in the middle. We refer to these slices as <u>washers</u>

volume of a washer
$= (\pi R^2 - \pi r^2) dx$



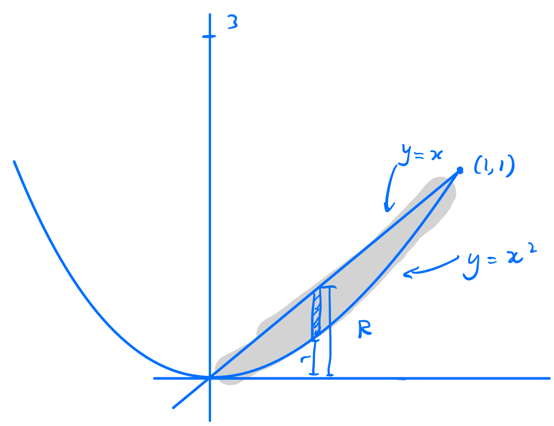
Example Revolve the region bounded by

$y = x$ and $y = x^2$ around

① $y = 0$

② $y = 3$.

Find the volume.

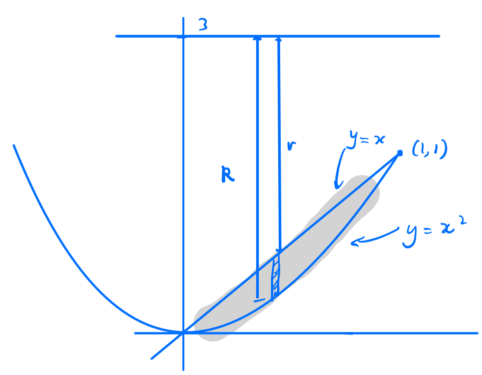


① $R(x) = x$

$r(x) = x^2$

$$V = \int_0^1 \pi\left(R(x)^2 - r(x)^2 \right) dx$$

$$= \int_0^1 \pi\left(x^2 - (x^2)^2 \right) dx$$



② $R(x) = 3 - x^2$

$r(x) = 3 - x$

$$V = \int_0^1 \pi\left((3-x^2)^2 - (3-x)^2 \right) dx$$

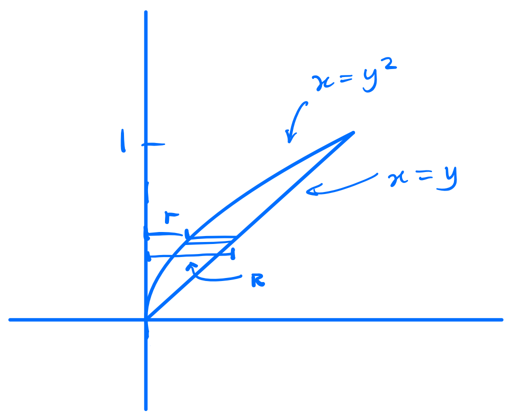
<u>Example</u> Revolve the region bounded by $y=\sqrt{x}$ and $y=x$ around

① $x=0$

② $x=1$

and find the volume.

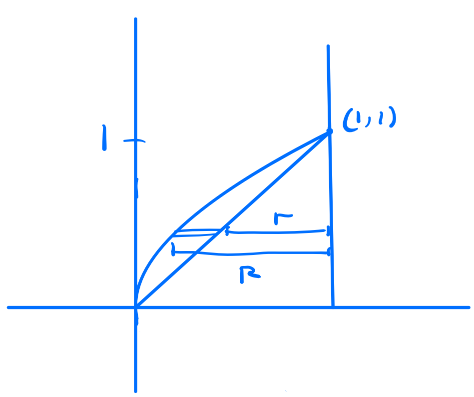
①



$R(y)=y, \quad r(y)=y^2$

$$V=\int_0^1 \pi\left(R(y)^2-r(y)^2\right)dy$$

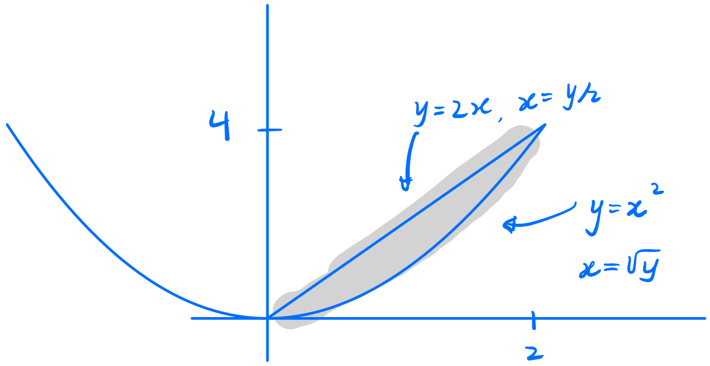
$$=\int_0^1 \pi\left(y^2-(y^2)^2\right)dy$$



$R(y)=1-y^2, \quad r(y)=1-y$

$$V=\int_0^1 \pi\left((1-y^2)^2-(1-y)^2\right)dy$$

Problem 3. Consider the region R bounded by $y = 2x$ and $y = x^2$. Use the washer method to set up (1) an integral for the volume when R is revolved around the x-axis, and (2) an integral for the volume when R is revolved around the y-axis.



$$x^2 = 2x$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, 2$$

(1) $R(x) = 2x, \quad r(x) = x^2$

$$\int_0^2 \pi \left((2x)^2 - (x^2)^2 \right) dx$$

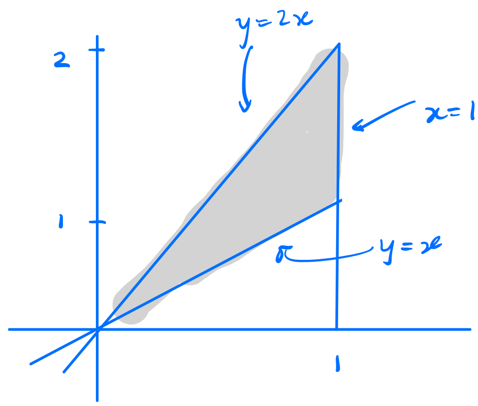
(2) $R(y) = \sqrt{y} \quad , \quad r(y) = y/2$

$$\int_0^4 \pi \left((\sqrt{y})^2 - (y/2)^2 \right) dy$$

Problem 2. Use the washer method to set up (1) an integral for the volume when the region bounded by

$$y = x, y = 2x, x = 1$$

is revolved around the x-axis, and (2) an integral for the volume when the region below is revolved around the y-axis.



(1) $R(x) = 2x$, $R(x) = x$

$$\int_0^1 \pi\left((2x)^2 - x^2\right) dx$$

(2) lower half $(y \leq 1)$:

$R(y) = y$, $r(y) = y/2$

upper half $(1 \leq y \leq 2)$

$R(y) = 1$, $r(y) = y/2$

$$\int_0^1 \pi\left(y^2 - (y/2)^2\right) dy + \int_1^2 \pi\left(1^2 - (y/2)^2\right) dy$$

Problem 1. Use the washer method to set up an integral to find the volume of the solid of revolution formed by revolving the region bounded between $y = 4 - x^2$, $y = 0$, and $x = 0$ around the axes below.

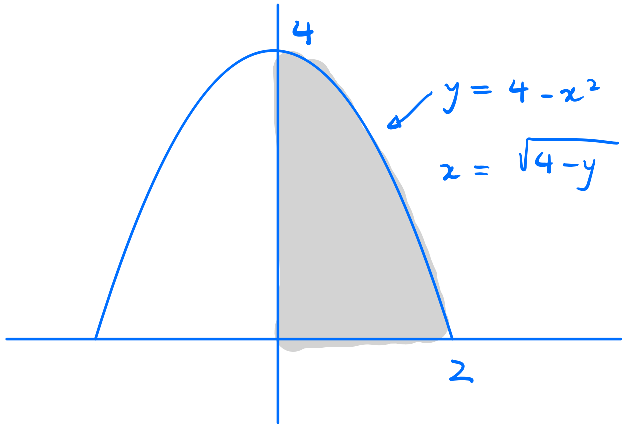
} addendum: in first quadrant

a. $y = -1$.

b. $y = 5$.

c. $x = -1$.

d. $x = 3$.



(a) $R(x) = (4 - x^2) - (-1) = 5 - x^2$

$r(x) = 1$

$$\int_0^2 \pi \left((5-x^2)^2 - 1^2 \right) dx$$

(b) $R(x) = 5$

$r(x) = 5 - (4-x^2) = 1 + x^2$

$$\int_0^2 \pi \left(4^2 - (1+x^2)^2 \right) dx$$

(c) $R(y) = \sqrt{4-y} - (-1) = \sqrt{4-y} + 1$

$r(y) = 0 - (-1) = 1$

$$\int_0^4 \pi \left((\sqrt{4-y} + 1)^2 - 1^2 \right) dy$$

(d) $R(y) = 3 - 0 = 3$

$r(y) = 3 - \sqrt{4-y}$

$$\int_0^4 \pi \left(3^2 - (3 - \sqrt{4-y})^2 \right) dy$$