§7.2 Integration by parts

How do we work with the following integrals:

$$\int x e^x \, dx , \qquad \int x \cos x \ ?$$

Recall the product rule for derivatives:

$$\frac{d}{dx}\Big(f(x) g(x) \Big) = f'(x) g(x) + f(x) g'(x)$$

or

$$(uv)' = u'v + uv'$$

$$= v\,du + u\,dv \quad \text{(shorthand notation)}$$

By integrating both sides:

$$\int (uv)' = \int v\,du + \int u\,dv.$$

Notice $\int (uv)' = uv$. So

$$uv = \int v\,du + \int u\,dv.$$

Rearrange this equation, we get

Integration by parts formula

$$\int u\,dv = uv - \int v\,du$$

Examples

① $\int x e^x \, dx$

$u = x \qquad dv = e^x \, dx$

$du = dx \qquad v = e^x$

$= xe^x - \int e^x \, dx$

$= xe^x - e^x + C$

Comments

① we're free to choose u, dv

② we often pick u to be the factor that gets "simpler" when you take its derivative

③ we often pick dv to be a function we can integrate

(2) $\displaystyle\int x\cos x\,dx$

$u = x \qquad dv = \cos x\,dx$

$du = dx \qquad v = \sin x\,dx$

$= x\sin x - \displaystyle\int \sin x\,dx$

$= x\sin x + \cos x + C$

(3) $\displaystyle\int x^6 \ln x\,dx$

$u = \ln x \qquad dv = x^6\,dx$

$du = \frac{1}{x} \qquad v = \frac{1}{7}x^7$

$= \frac{1}{7}x^7 \ln x - \frac{1}{7}\displaystyle\int x^6\,dx$

$= \frac{1}{7}x^7 \ln x - \frac{1}{49}x^7 + C$

(4) $\displaystyle\int_2^3 \ln x\,dx$

$= \displaystyle\int_2^3 1 \cdot \ln x\,dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$= x\ln x \Big|_2^3 - \displaystyle\int_2^3 dx$

$= 3\ln 3 - 2\ln 2 - 1$