

§ 8.2 Volumes of revolution

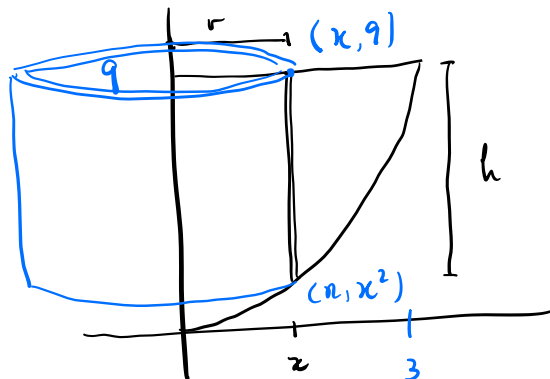
Let's discuss another method for finding volumes of revolution called the shell method.

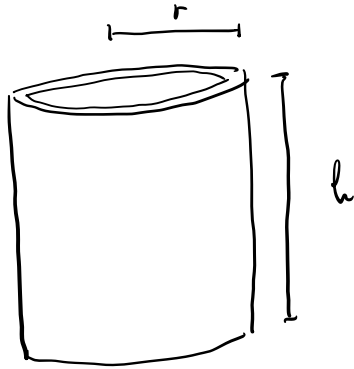
Example Revolve the region between

$$y = x^2 \quad \text{and} \quad y = 9 \quad \text{for } x \text{ values}$$

between 0 and 3 about the y -axis.

Find its volume using shells.





$$r = x, \quad h = 9 - x^2$$

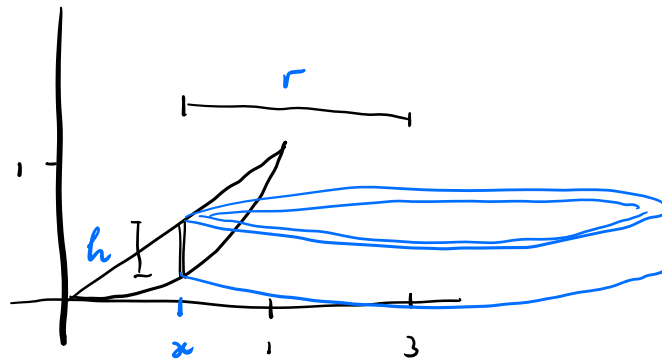
Total volume

$$V = \int_0^3 2\pi r h dx$$

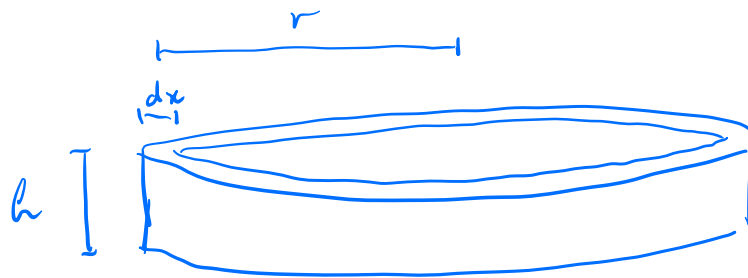
$$= \int_0^3 2\pi x (9 - x^2) dx$$

Compare the value of this integral
with Wednesday worksheet problem 2a.

Example Find the volume of the solid formed by rotating the region bounded by $y=x$ and $y=x^2$ about the line $x=3$.



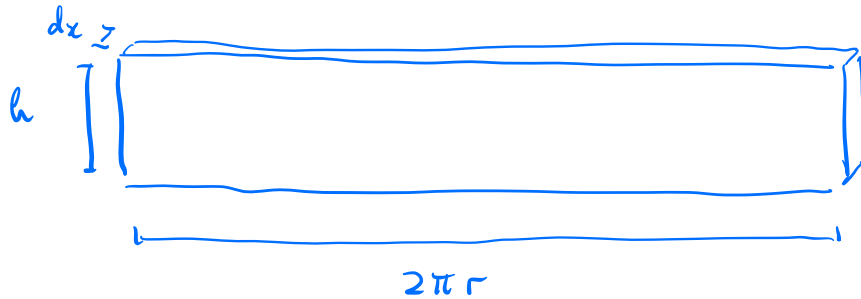
What do we get when rotating a horizontal slice about $y=3$?



$$r = 3 - x, \quad h = x - x^2$$

What is the volume of this shell?

If we cut it and unfold it, we get



$$\begin{aligned}\text{volume of shell} &= 2\pi r h dx \\ &= 2\pi (3-x)(x-x^2) dx\end{aligned}$$

$$\text{total volume} = \int_0^1 2\pi (3-x)(x-x^2) dx$$

Key rules

- In disc/washer method,
slices are perpendicular to
axis of revolution.
- In shell method,
slices are parallel to
axis of revolution.
- vertical slices use x -variables
- horizontal slices use y -variables.

