

Math 102 — Series review

Summary. This review sheet is meant to get you to review the tests we've learned so far. You'll rewrite the statement of each test, and give one or two examples of series that the test can be applied to (and possibly one or two examples where the test is inconclusive).

0.1 n th term test

Consider the series $\sum a_n$.

Statement of test:

Examples:

Inconclusive examples:

0.2 Geometric series

Consider a series of the form

$$\sum_{n=0}^{\infty} ax^n = a + ax + ax^2 + ax^3 + \dots$$

Statement of convergence/divergence criteria:

Convergent examples:

Divergent examples:

0.3 p -series

Consider a series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \dots$$

Statement of convergence/divergence criteria:

Convergent examples:

Divergent examples:

0.4 Comparison test

Suppose $\sum a_n$ and $\sum b_n$ are series where

$$0 \leq a_n \leq b_n$$

for all n .

Statement of test:

Convergent examples:

Divergent examples:

0.5 Limit comparison test

Suppose $\sum a_n$ and $\sum b_n$ are series where $a_n \geq 0$ and $b_n \geq 0$ for all n .

Statement of test:

Convergent examples:

Divergent examples:

0.6 Absolute convergence test

Consider the series $\sum a_n$.

Statement of test:

Examples:

Inconclusive examples:

0.7 Ratio test

Consider the series $\sum a_n$ and let

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

Statement of test:

Examples:

Inconclusive examples:

0.8 Alternating series test

Consider the series $\sum(-1)^n b_n$ or $\sum(-1)^{n+1} b_n$ where $b_n \geq 0$ for all n .

Statement of test:

Conditionally convergent examples: