Math 102 — Integral basics

Summary. Try each of the following problems together in a small group.

Problem 1. For each of the following functions $f(x)$, find an antiderivative $F(x)$. That is, find a function $F(x)$ so that $F'(x) = f(x)$.

a. $f(x) = 5x - \sqrt{x} + \frac{1}{x^4}$

b. $f(x) = x^4 + \frac{1}{x}$

c. $f(x) = 2 + \cos x$

Problem 2. Find the following using antiderivatives and the Fundamental Theorem of Calculus.

a. $\int_0^5 (x^3 - x^2 + 10x)\,dx$

b. $\int_0^{\pi/4} \sin\theta\,d\theta$

c. $\int_0^1 4e^z\,dz$

d. $\int_1^2 \frac{1+y^2}{y}\,dy$

Problem 3. Suppose that $G(x)$ is an antiderivative of $g(x)$, and $G(4) = 9, G(6) = 4, G(9) = 6$. Find the following integrals.

a. $\int_4^6 g(x)\,dx + \int_6^9 g(x)\,dx$

b. $\int_4^9 (g(x) + 3)\,dx$

c. $\int_6^9 7g(x)\,dx$

Problem 1

(a) $F(x) = \frac{5}{2}x^2 - \frac{2}{3}x^{3/2} - \frac{1}{3}x^{-3}$

(b) $F(x) = \frac{1}{5}x^5 + \ln|x|$

(c) $F(x) = 2x + \sin x$

Problem 2

(a) $\frac{1}{4}x^4 - \frac{1}{3}x^3 + 5x^2 \Big|_0^5$

$= \frac{1}{4}(5)^4 - \frac{1}{3}(5)^3 + 5(5)^2$

(b) $-\cos\theta \Big|_0^{\pi/4} = -\left(\frac{\sqrt{2}}{2} - 1\right)$

$= -\frac{\sqrt{2}}{2} + 1$

(c) $4e^x \Big|_0^1 = 4e - 4$

(d) $$\int_1^2 \left(\frac{1}{y} + \frac{y^2}{y}\right) dy = \int_1^2 \left(\frac{1}{y} + y\right) dy$$

$$= \ln|y| + \frac{1}{2}y^2 \Big|_1^2$$

$$= \ln 2 + 2 - \frac{1}{2}$$

$$= \ln 2 + \frac{3}{2}$$

Problem 3

(a) $$\int_4^9 g(x)\,dx = G(9) - G(4) = 6 - 9 = -3$$

(b) $$\int_4^9 g(x) + \int_4^9 3\,dx$$

$$= G(9) - G(4) + 15$$

$$= -3 + 15 = 12$$

(c) $$\int_6^9 7\,g(x)\,dx = 7\int_6^9 g(x)\,dx$$

$$= 7\left(G(9) - G(6)\right) = 7(6-4) = 14$$