Math 102 — Finding areas

Summary. Try each of the following problems together in a small group.

Problem 1. Consider the region between $y = e^{-x}$ and $y = -x^2$ for values of x between 0 and 1.

a. Sketch a picture of the region.

b. Which is the top curve and which is the bottom curve?

c. Set up and compute the integral that represents the area of the region.

Problem 2. Consider the region bounded the curves $y = x^2$ and $y = 8 - x^2$.

a. Where do these two curves intersect? That is, for which x-values do we have $x^2 = 8 - x^2$?

b. Sketch a picture of the region. Which is the top curve and which is the bottom curve?

c. Set up and compute the integral that represents the area of the region.

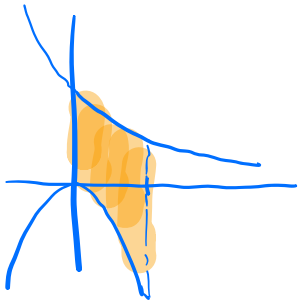
Problem 3. Consider the bounded region between the x-axis and the graph of $y = x^2 - x$.

a. Where does $y = x^2 - x$ intersect the x-axis? That is, for which x-values do we have $x^2 - x = 0$?

b. Sketch a picture of the region. Which is the top curve and which is the bottom curve?

c. Set up and compute the integral that represents the area of the region.

Problem 1



$$\int_0^1 \left[e^{-x} - (-x^2)\right] dx$$

$$= \int_0^1 \left(e^{-x} + x^2\right) dx$$

$$= -e^{-x} + \frac{1}{3}x^3 \Big|_0^1$$

$$= \left[-e^{-1} + \frac{1}{3}\right] - \left[-e^{0} + 0\right]$$

$$= \frac{4}{3} - e^{-1}$$

Note $\int e^{-x}\, dx = -e^{-x}$

since $\frac{d}{dx}\left(-e^{-x}\right) = -\frac{d}{dx}\left(e^{-x}\right)$ (pull constant out)

$= -e^{-x} \cdot (-1)$ (chain rule)

$= e^{-x}$ (simplify)

Problem 2

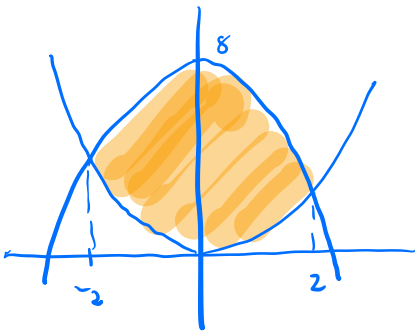
The curves intersect when

$$x^2 = 8 - x^2$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \pm 2$$



$$\int_{-2}^{2} \left[(8 - x^2) - (x^2) \right] dx$$

$$= \int_{-2}^{2} (8 - 2x^2)\, dx$$

$$= 8x - \frac{2}{3}x^3 \Big|_{-2}^{2}$$

$$= \left(16 - \frac{2}{3}(8)\right) - \left(-16 - \frac{2}{3}(-8)\right)$$

$$= 32 - \frac{4}{3}(8)$$

$$= 32 - \frac{32}{3} = \frac{96 - 32}{3} = \frac{64}{3}$$

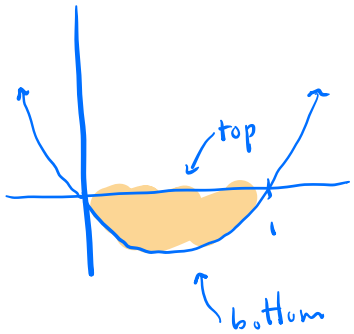
Problem 3

The curve intersects the x-axis when

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0, 1$$



Area

$$= \int_0^1 \left[\underbrace{(0)}_{\text{top}} - \underbrace{(x^2 - x)}_{\text{bottom}} \right]$$

$$= \int_0^1 (-x^2 + x)\, dx$$

$$= -\frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_0^1$$

$$= -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$$