Math 102 — Geometric series

Problem 1. For each of the following, determine whether it is a geometric series. If it is, state the first term, the common ratio between success terms, and whether it converges. If it converges, find the value it converges to; that is, find its sum.

a. $5 - 10 + 20 - 40 + 80 - \cdots$

b. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \cdots$

c. $\frac{1}{3} + \frac{4}{9} + \frac{16}{27} + \frac{64}{81} + \cdots$

d. $\sum_{n=0}^{\infty} \frac{5}{3^{3n}}$

e. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{2^n}$

f. $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^{2n+2}}$

g. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Problem 2. For each of the following geometric series, find a formula for the sum in terms of z and find the values of z, expressed as an interval, for which the series converges.

a. $\sum_{n=0}^{\infty} 5^n z^n$

b. $z/3 - z^2/3 + z^3/3 - z^4/4 + z^5/5 - \cdots$

c. $\sum_{n=2}^{\infty} (z-4)^n$

d. $8 + 8(6-z) + 8(6-z)^2 + 8(6-z)^3 + \cdots$

Problem 3 (Bonus challenge problem)**.** The Cantor set is a fractal that is formed on the real line through the following iterative process. We start with the closed unit interval $[0,1]$ and in the first step of the process remove the open interval $C_1 = (1/3, 2/3)$. In the second step, we take the remaining two closed intervals, $[0, 1/3]$ and $[2/3, 1]$, and remove each of their respective middle thirds, $C_2 = (1/9, 2/9) \cup (7/9, 8/9)$. What remains is 4 closed intervals and from each of them, we remove their 4 respective middle thirds, which we denote as C_3. The Cantor set is what remains after repeating this removal process *ad infinitum,* ie. after removing $C_1, C_2, C_3, C_4, C_5, \ldots$.

a. What is the total length of all the middle-thirds intervals that are removed?

b. What are some elements of the Cantor set? That is, what are some real numbers that never get removed from the process? (It turns out there is an *uncountable* number of elements in the Cantor set.)