Math 102 — More partial fractions

Summary. Try each of the following problems together in a small group.

Problem 1. To get started on Homework 8, try each of the following problems.

a. $\int \frac{20}{25-x^2}\,dx$

b. $\int \frac{2(1+x)}{x(x^2+3x+2)}\,dx$

c. $\int \frac{x-2}{x^2-x^4}\,dx$

d. $\int \frac{x^2-5}{x(x^2+1)}\,dx$

Problem 2. Here are a couple problems that offer a new idea called trigonometric substitution. It's a cool, relatively straightforward idea, but there are more important topics to get to, so try these just for fun if you have time. These won't appear on homework, quizzes, or exams. Ask me questions when you get stuck!

a. Find $\int \frac{1}{x^2+9}\,dx$ by doing the substitution $x = 3\tan\theta$. When finding dx, remember that the derivative of $\tan\theta$ is $\sec^2\theta$.

b. Find $\int \frac{1}{\sqrt{4-x^2}}\,dx$ by doing the substitution $x = 2\sin\theta$. Remember to find dx and substitute it in as well.

Problem 1 there are homework

Problem 2 solutions

a

$$x = 3\tan\theta$$

$$dx = 3\sec^2\theta\, d\theta$$

$$\int \frac{1}{x^2+9}\,dx = \int \frac{1}{9\tan^2\theta + 9}\, 3\sec^2\theta \, d\theta$$

$$= \frac{1}{3}\int \frac{1}{\tan^2\theta + 1} \cdot \frac{1}{\cos^2\theta}\, d\theta$$

$$= \frac{1}{3}\int \frac{1}{\frac{\sin^2\theta}{\cos^2\theta} + 1} \quad \frac{1}{\cos^2\theta}\, d\theta$$

$$= \frac{1}{3}\int \frac{1}{\sin^2\theta + \cos^2\theta}\, d\theta$$

$$= \frac{1}{3}\int d\theta$$

$\tan\theta = \frac{x}{3}$

$$= \frac{1}{3}\theta + C$$

$\theta = \arctan\left(\frac{x}{3}\right)$

$$= \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

(b) $x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$$\int \frac{1}{\sqrt{4 - x^2}}\, dx$$

$$= \int \frac{1}{\sqrt{4 - 4\sin^2\theta}}\, 2\cos\theta \, d\theta$$

$$= \int \frac{1}{\sqrt{4(1-\sin^2\theta)}}\, 2\cos\theta \, d\theta$$

$$= \int \frac{1}{\sqrt{4\cos^2\theta}} \, 2\cos\theta \, d\theta$$

$$= \int \frac{1}{2\cos\theta} \, 2\cos\theta \, d\theta$$

$$= \int d\theta$$

$$= \theta + C$$

$$= \arcsin\left(\frac{x}{2}\right) + C$$