Math 102 — Volumes of revolution

Summary. Try each of the following problems together in a small group.

Problem 1. Use slices to set up an integral to find the volume of the solid formed by revolving each of the following regions around the given axis. If you have time at the end, try computing the integrals, but first focus on setting them all up.

a. Bounded by: $y = e^x, y = 0, x = -1, x = 1$. Revolved around: x-axis.

b. Bounded by: $y = 4 - x^2, y = 0, x = -2, x = 0$. Revolved around: x-axis.

c. Bounded by: $y = e^{-x}, y = 0, x = 0, x = 1$. Revolved around: $y = 2$.

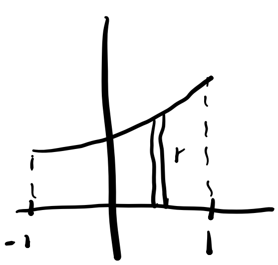
d. Bounded by: $y = x^2, y = x, x = 0, x = 1$. Revolved around: $y = -2$.

Problem 2. Just like in the last problem, use slices to set up an integral to find the volume of the solid formed by revolving each of the following regions around the given axis. In our examples so far, our axis of revolution has always been horizontal, but the following examples are new since they have a vertical axis of revolution. Try making your slices horizontally and use y-variables instead of x-variables. This is a slightly new idea, but you're ready to give it a shot.

a. Bounded by: $y = x^2, y = 9, x = 0, x = 3$. Revolved around: y-axis.

b. Bounded by: $y = 2x, y = 0, x = 0, x = 3$. Revolved around: $x = 3$.

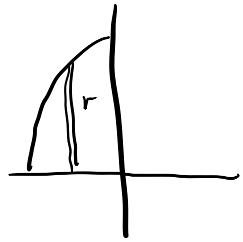
(a)



$r = e^x$

$V = \int_{-1}^{1} \pi r^2 \, dx = \int_{-1}^{1} \pi e^{2x} \, dx$

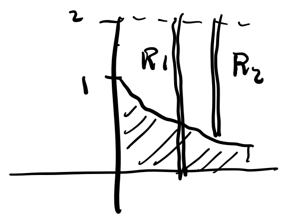
(b)



$r = 4 - x^2$

$$V = \int_{-2}^{0} \pi r^2 \, dx = \int_{-2}^{0} \pi (4 - x^2)^2 \, dx$$

(c)



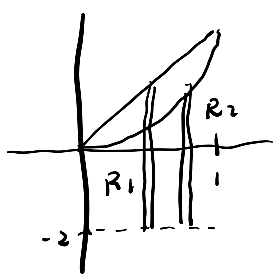
$R_1 = 2 - 0$

$R_2 = 2 - e^{-x}$

$$V = \int_0^1 (\pi R_1^2 - \pi R_2^2) \, dx$$

$$= \int_0^1 \pi \left(4 - (2 - e^{-x})^2\right) dx$$

(d)



$R_1 = x - (-2) = x + 2$

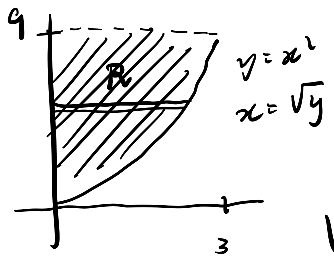
$R_2 = x^2 - (-2) = x^2 + 2$

$$V = \int_0^1 (\pi R_1^2 - \pi R_2^2) dx$$

$$= \int_0^1 \pi \left((x+2)^2 - (x^2+2)^2 \right) dx$$

Problem 2

(a)

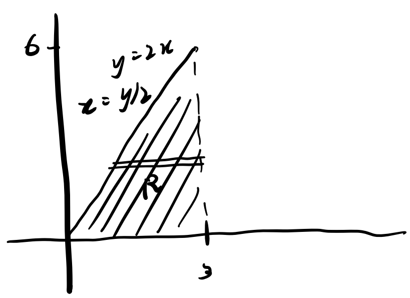


$$R = \sqrt{y}$$

$$V = \int_0^9 \pi R^2 \, dy$$

$$= \int_0^9 \pi (\sqrt{y})^2 \, dy$$

(b)



$$R = 3 - \frac{y}{2}$$

$$V = \int_0^6 \pi R^2 \, dy$$

$$= \int_0^6 \pi \left(3 - \frac{y}{2}\right)^2 dy$$