Math 102 — Exam 2 review

Your exam in class on April 18 will contain about 7 problems, some with multiple parts. It will cover material from Homework 5 to Homework 8. The problems below give you a sampling of some similar problems, but it's not necessarily comprehensive, so make sure to review old homework, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but you can use a scientific calculator with no graphing functionality.

Problem 1. Sketch the region bounded by the given curves and then set up integrals to find the volume obtained by rotating the region about the given axis. Consider only the portion of the region in the 1st quadrant if it spans multiple quadrants.

a. $x + y = 1, \quad y = 0, \quad x = 0;$ about the x-axis

b. $y = x^2, \quad y = x;$ about the y-axis

c. $y = x^2, \quad y = 9, \quad x = 0;$ about the x-axis

d. $y = x^2, \quad y = 9, \quad x = 0;$ about the y-axis

Problem 2. A variety of tanks filled with oil, which has density 800 kg per cubic meter, are given below. Each tank is only filled to half the tank's height. Set up but do not compute an integral for the work performed in pumping all the oil to a height 3 meters above the top of the tank.

a. An upright circular cylinder with height 4 m and base radius 3 m.

b. A cone whose base circle has radius 3 m and whose height is 10 m, oriented so that its tip is at the bottom of the tank.

c. A pyramid whose base is a square with side length 4 m and whose height is 7 m, oriented so that its tip is at the bottom of the tank.

Problem 3. First, consider a_n below as the nth term of a sequence. State whether the sequence converges and, if so, find its limit. Second, consider a_n as the nth term of a series. State whether the series converges and, if possible, find its sum.

a. $a_n = \frac{n^3+4n^2+3}{8n^4+5n+7}$

b. $a_n = \frac{9^{n+1}}{10^n}$

c. $a_n = \frac{n^4+1}{3n^4+4}$

d. $a_n = n^2 2^{-n}$

e. $a_n = 7$

Problem 4. Find the sum of the infinite series

a. $-3 + 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots$

b. $\displaystyle\sum_{n=1}^{\infty} 6(0.9)^{n-1}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{4^n}$

Problem 5. Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges where a_n is given below. State the test used and make sure to justify your use of the test with appropriate details.

a. $a_n = \frac{1}{n\sqrt{n^2+1}}$

b. $a_n = \frac{n!}{5^n}$

c. $a_n = \frac{n}{3n+1}$

d. $a_n = (-1)^n \frac{1}{5n^2+1}$

e. $a_n = (-1)^{n-1} \frac{1}{\sqrt{5n^2+1}}$

f. $a_n = \frac{3}{n+(1.2)^n}$

g. $a_n = \frac{2^n n!}{(n+2)!}$

h. $a_n = \frac{n^{0.1}-1}{n(\sqrt{n}+1)}$

Problem 6. Determine whether the following series converge absolutely, converge conditionally, or diverge.

a. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^5+2}$

b. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{5n+2}}$

c. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{1/3}+3}$