

Math 102 — Taylor series

Problem 1. Write out the first four non-zero terms of the Taylor series of f centered at c for the following examples.

a. $f(x) = \sin x, c = \pi/2$

b. $f(x) = \sqrt{x}, c = 4$

c. $f(x) = \ln x, c = 2$

Problem 2. Listed below are the Taylor series about 0 of some common functions. Find the general formula for the n th term and express the series with summation notation.

a. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 - \dots$

b. $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \dots$

c. $\arctan x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$

Problem 3. Find the sum of the following series by recognizing they are given by substituting a constant into a known Taylor series.

a. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

b. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} 9^n$

c. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}$

d. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{2^{2n+1}(2n+1)!}$