

## Math 102 — More on Taylor series

*Summary.* Try each of the following problems together in a small group.

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**Problem 1.** Use the Taylor series for  $\sin x$ ,  $\cos x$ ,  $e^x$ , or  $\frac{1}{1-x}$  to find the first 4 terms of the Taylor series for the following functions.

- $f(x) = \ln(1+x)$  (Hint: integrate the terms of a known Taylor series)
- $f(x) = \frac{1}{(1-x)^3}$  (Hint: differentiate a known Taylor series)
- $f(x) = x^2 \sin x^3$
- $f(x) = xe^{-x^2}$

**Problem 2.** Use the first 4 terms of the Taylor series for  $\sin x$ ,  $\cos x$ ,  $e^x$ , or  $\ln(1+x)$  to approximate the following numbers.

- $\ln(1.5)$
- $e$
- $\frac{\sqrt{2}}{2}$
- $\sqrt{3}$

**Problem 3.** Consider the function  $f(x) = e^{-x^2}$ .

- Find the first three terms of the Taylor series of  $f$ .
- Use these first three terms to approximate the integral

$$\int_0^1 e^{-x^2} dx$$

**Problem 4.** Use a known Taylor series to compute the following values.

- $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$
- $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$