

Math 102 — Exam 3 review

Your self-scheduled exam during the exam period May 9-13 will contain 7 problems, some with multiple parts. It will mostly cover material from Homework 9 and 10 as well as material from the last full week of class. Five problems will be on this new material, which comes from textbook section 9.5 and sections 10.1-10.4. There will also be 1 problem similar to a problem on Exam 1 and 1 problem similar to a problem on Exam 2. The problems below give you a sampling of some similar problems from the new material, but it's not necessarily comprehensive, so make sure to review old homework, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but I have provided you with the general Taylor series formula and the error estimate expression in Taylor's Theorem. You may also use a scientific calculator with no graphing functionality.

Problem 1. Find the interval of convergence of the following power series.

a.
$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$$

b.
$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n 4^n}$$

c.
$$\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$$

Problem 2. Find the first 5 terms of the Taylor series about 0 of each of the following functions and then write the full series using summation notation.

a. $f(x) = \sin(x^4)$

b. $f(x) = x e^{2x}$

c. $f(x) = \ln(1 - x^3)$

d. $f(x) = \frac{x^3}{(1+x)^2}$

e. $f(x) = x^4 + 4x^3 + 5x^2 - 3x - 7$

Problem 3. Find the sum of the series below using known Taylor series.

a.
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n! 4^n}$$

b.
$$\sum_{n=0}^{\infty} (-1/4)^n$$

c.
$$\sum_{n=1}^{\infty} n (-1/4)^{n-1}$$

d.
$$\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{4^{2n+1} (2n+1)!}$$

Problem 4. Use Taylor's theorem to find a bound for the error in approximating the given quantity with a third degree Taylor polynomial about 0 for the given function.

a. $\sin(0.2)$, $f(x) = \sin x$

b. $\sqrt{0.9}$, $f(x) = \sqrt{1+x}$

c. $1/\sqrt{3}$, $f(x) = (1+x)^{-1/2}$

Problem 5. Find the Taylor polynomial of degree 4 of $f(x)$ about c for the given examples below.

a. $f(x) = 1/(1+x)$, $c = 2$

b. $f(x) = \sin x$, $c = -\pi/4$

c. $f(x) = \ln(x^2)$, $c = 1$