Math 102 — Comparison test

Summary. Try each of the following problems together in a small group.

Problem 1. Make a conjecture about whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{1}{n^2+2}$ conv. $p=2$

b. $\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$ conv. $p=2$

c. $\sum_{n=1}^{\infty} \frac{1}{n+e^n}$ conv. geom

d. $\sum_{n=1}^{\infty} \frac{n}{5n^2-3}$ div. $p=1$

Problem 2. Use the comparison test to explain why your conjectures are true.

a) Note $n^2+2 \geq n^2$, so $\frac{1}{n^2+2} \leq \frac{1}{n^2}$.

Therefore $\sum \frac{1}{n^2+2}$ converges by comparison test since $\sum \frac{1}{n^2}$ converges

b) Note $n^4+1 \geq n^4$, so $\frac{n^2}{n^4+1} \leq \frac{n^2}{n^4} = \frac{1}{n^2}$

Therefore $\sum \frac{n^2}{n^4+1}$ converges by comp. test since $\sum \frac{1}{n^2}$ converges.

c) Note $n+e^n \geq e^n$, so $\frac{1}{n+e^n} \leq \frac{1}{e^n}$.

Therefore $\sum \frac{1}{n+e^n}$ converges by comparison test since $\sum \frac{1}{e^n}$ converges (it's a geometric series with $|x| = \frac{1}{e} < 1$.)

d) Note $5n^2-3 \leq 5n^2$, so $\frac{n}{5n^2-3} \geq \frac{n}{5n^2} = \frac{1}{5n}$

Therefore $\sum \frac{n}{5n^2-3}$ diverges by comp. test since $\sum \frac{1}{5n}$ diverges.