Math 102 — Limit comparison test

Summary. Try each of the following problems together in a small group.

Problem 1. Make a conjecture about whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{n^3+7}{n^4-6}$ diverges $p=1$

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{7n^5+3n^2-2}}$ converges $p=5/2$

c. $\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{7n^7+3n^2-2}}$ diverges $p=1/2$

d. $\sum_{n=1}^{\infty} \frac{n^2}{\sin n+n^3}$ diverges $p=1$

Problem 2. Use the limit comparison test to explain why your conjectures are true.

(a) Let $b_n = \frac{1}{n}$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^3+7}{n^4-6}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^4+7n}{n^4-6} = 1 > 0$$

Then $\sum a_n$ <u>diverges</u> since $\sum b_n$ diverges by LCT.

(b) Let $b_n = \frac{1}{n^{5/2}}$. Then

$$\lim \frac{a_n}{b_n} = \lim \frac{n^{5/2}}{\sqrt{7n^5+3n^2-2}} = \lim \sqrt{\frac{n^5}{7n^5+3n^2-2}}$$

$$= \sqrt{\frac{1}{7}} > 0$$

Then $\sum a_n$ converges since $\sum b_n$ converges.

by LCT.

(c) Let $b_n = \frac{1}{n^{1/2}}$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^{3.5}}{\sqrt{7n^7 + 3n^2 - 2}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n^7}{7n^7 + 3n^5 - 2}}$$

$$= \sqrt{\frac{1}{7}} > 0$$

Then $\sum a_n$ diverges since $\sum b_n$ diverges

by LCT.

(d) Let $b_n = \frac{1}{n}$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3}{\sin n + n^3} = 1 > 0$$

Then $\sum a_n$ diverges since $\sum b_n$ diverges

by LCT.