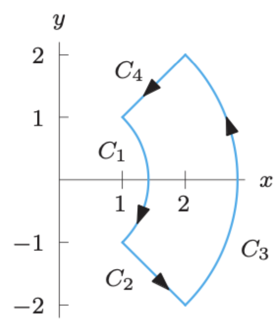
Math 203, Spring 2023 — Homework 10

Tim Chumley

Due April 28

Instructions. This problem set has material from Week 13 of class.

Problem 1. Let $\mathbf{F}(x, y) = \langle x, y \rangle$ and consider the closed curve $C = C_1 + C_2 + C_3 + C_4$ shown below, with arrows indicating the orientation of each component C_1, C_2, C_3, C_4. Notice that C_1 and C_3 are arcs of circles centered at the origin and C_2 and C_4 are radial line segments.. Indicate the sign (positive, negative, or zero) of each of the following line integrals.



Line integral	Sign
$\int_{C_1} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\int_{C_2} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\int_{C_3} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\int_{C_4} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\oint_{C} \mathbf{F} \cdot \mathbf{n}\, ds$	

Problem 2. Repeat Problem 1 using $\mathbf{F}(x, y) = \langle -y, x \rangle$.

Line integral	Sign
$\int_{C_1} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\int_{C_2} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\int_{C_3} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\int_{C_4} \mathbf{F} \cdot \mathbf{n}\, ds$	
$\oint_{C} \mathbf{F} \cdot \mathbf{n}\, ds$	

Problem 3. For each given vector field $\mathbf{F}$ and oriented curve C, compute $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$ by computing $\int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \langle g'(t), -f'(t) \rangle\, dt$ where $\mathbf{r}(t) = \langle f(t), g(t) \rangle, a \leq t \leq b$, is a parametrization of C. You can use Wolfram Alpha to compute the integral, but make sure to give a numerical final answer, not just a setup of the integral.

a. $\mathbf{F}(x, y) = \langle x + y, x - y \rangle$, C is the line segment oriented from $(3, -2)$ to $(3, 2)$.

b. $\mathbf{F}(x, y) = \langle x^2, y + 1 \rangle$, C is the portion of the parabola $y = x^2$ oriented from $(0, 0)$ to $(2, 4)$.

Problem 4. For each given vector field $\mathbf{F}$ and oriented curve C, compute $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$ by computing $\int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \langle g'(t), -f'(t) \rangle\, dt$ where $\mathbf{r}(t) = \langle f(t), g(t) \rangle, a \leq t \leq b$, is a parametrization of C. You can use Wolfram Alpha to compute the integral, but make sure to give a numerical final answer, not just a setup of the integral.

a. $\mathbf{F}(x, y) = \langle y, 0 \rangle$, C is the line segment from $(0, 0)$ to $(3, 3)$.

b. $\mathbf{F}(x, y) = \langle y, x \rangle$, C is the portion of the curve $x = y^3$ from $(-8, -2)$ to $(8, 2)$.

Problem 5. For each given vector field $\mathbf{F}$ and piecewise smooth, closed, positively oriented curve C, compute $\operatorname{div} \mathbf{F}$ and use the Divergence Theorem to compute $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$. You can use Wolfram Alpha to compute the integral, but make sure to give a numerical final answer, not just a setup of the integral.

a. $\mathbf{F}(x, y) = \langle x - y, x + y \rangle$, C is the closed curve composed of the parabola $y = 4 - x^2$ on $0 \leq x \leq 2$ followed by the line segment from $(2, 0)$ to $(0, 4)$.

b. $\mathbf{F}(x, y) = \langle -y, x \rangle$, C is the unit circle.

Problem 6. For each given vector field $\mathbf{F}$ and piecewise smooth, closed, positively oriented curve C, compute $\operatorname{div} \mathbf{F}$ and use the Divergence Theorem to compute $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$. You can use Wolfram Alpha to compute the integral, but make sure to give a numerical final answer, not just a setup of the integral.

a. $\mathbf{F}(x, y) = \langle 0, y^2 \rangle$, C is the triangle with corners at $(0, 0)$, $(0, 2)$, and $(1, 1)$.

b. $\mathbf{F}(x, y) = \langle x^2/2, y^2/2 \rangle$, C is the curve that starts at $(0, 1)$, follows the parabola $y = (x - 1)^2$ to $(3, 4)$, then follows a line back to $(0, 1)$.