

Math 203, Spring 2023 — Homework 3

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Due February 17

Instructions. This problem set has material from Week 3 of class.

Problem 1. Create a vector-valued function whose graph matches the given description.

- A circle of radius 3, centered at $(-6, 7)$ traced counter-clockwise for $0 \leq t \leq 2\pi$.
- A circle of radius 2, centered at $(1, 4)$ traced clockwise for $0 \leq t \leq 2\pi$.
- A line through the points $P = (7, 8, 9)$ and $Q = (11, 12, 10)$ with $\mathbf{r}(0) = P, \mathbf{r}(1) = Q$.
- A line through the points $P = (1, 3, 5)$ and $Q = (2, 4, 6)$ with $\mathbf{r}(0) = Q, \mathbf{r}(1) = P$.
- A vertically oriented helix that starts at $(2, 0, 0)$ at time $t = 0$ and ends at $(2, 0, 4)$ after one revolution at time $t = 2\pi$.

Problem 2. Sketch each vector-valued function on the interval $-2 \leq t \leq 2$ along with the tangent vectors $\mathbf{r}'(-1), \mathbf{r}'(0), \mathbf{r}'(1)$.

- $\mathbf{r}(t) = \langle t, t^2 + 2 \rangle, -2 \leq t \leq 2$
- $\mathbf{r}(t) = \langle t^2 + 1, t \rangle, -2 \leq t \leq 2$
- $\mathbf{r}(t) = \langle t, e^t \rangle, -2 \leq t \leq 2$

Problem 3. Give a vector equation for the tangent line of the graph of $\mathbf{r}(t)$ at the given t value.

- $\mathbf{r}(t) = \langle t^2 + t, t^2 - t \rangle, t = 1$
- $\mathbf{r}(t) = \langle 3 \cos t, \sin t \rangle, t = \pi/4$
- $\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, t \rangle, t = \pi$

Problem 4. Compute the derivatives of the following functions.

a. $\mathbf{r}(t) = \langle \cos(4t), \sin(t^2), e^{t^3+5} \rangle$

b. $\mathbf{r}(t) = \langle (t^3 + t^2)e^t, t^4 \sin t, t^5 \cos t \rangle$

c. $\mathbf{r}(t) = \left\langle \frac{t^2+t}{t^4+t^3}, \frac{\sin t}{t^4+1}, \frac{e^t}{t^2+t} \right\rangle$

Problem 5. Make a sketch by hand of each of the graphs of following functions. Use CalcPlot3d to sketch the graph as a check of your work (but no need to turn in the CalcPlot3d output).

a. $f(x, y) = 3 - (x^2 + y^2)$

b. $f(x, y) = (x - 1)^2 + (y - 1)^2$

c. $f(x, y) = -1 + \sqrt{x^2 + y^2}$

d. $f(x, y) = -\sqrt{(x - 1)^2 + (y - 1)^2}$

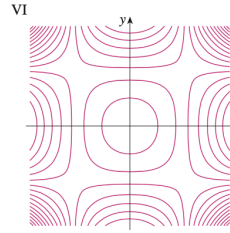
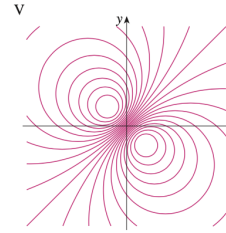
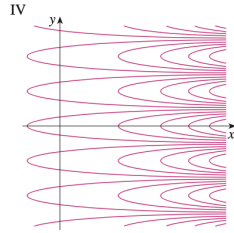
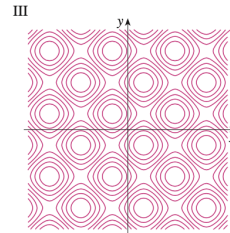
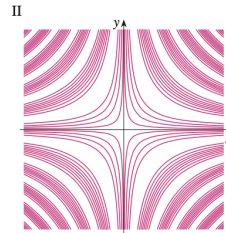
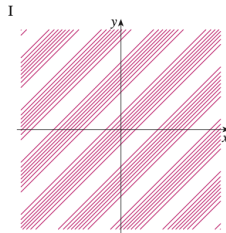
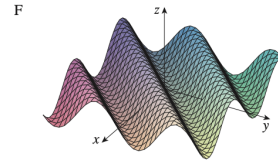
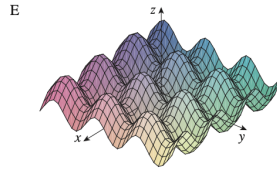
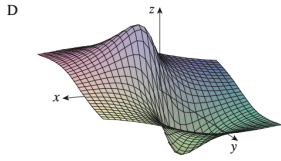
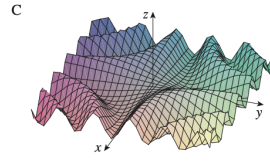
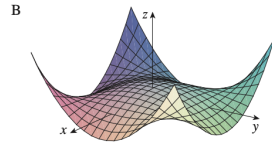
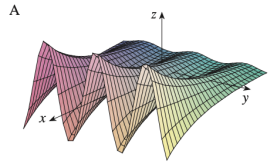
Problem 6. Make a contour plot for each of the following functions using the given c values. Make sure each curve is labeled with its corresponding c value. Use CalcPlot3d to sketch the graph of the function (but no need to turn in the output).

a. $f(x, y) = 4x + 2y, c = -2, 0, 2$

b. $f(x, y) = x - y^2, c = -2, 0, 2$

c. $f(x, y) = y^2, c = 0, 1, 2$

Problem 7. Shown below are 6 graphs of functions and their corresponding contour plots. Match them and organize your answer using a two column table in the given format.



Graph	Contour plot
A	
B	
C	
D	
E	
F	