

# Math 203, Spring 2023 — Homework 4

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Due March 3

**Instructions.** This problem set has material from Week 4 of class.

**Problem 1.** Compute  $f_x$ ,  $f_{xx}$ ,  $f_{xy}$ ,  $f_y$ ,  $f_{yy}$ , and  $f_{yx}$  for each function below.

a.  $f(x, y) = x^4y^3 + 5x^3y^2 - 8x^2y + 7xy^3$

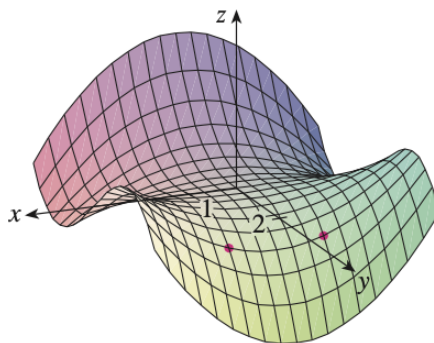
b.  $f(x, y) = 3 \sin(2x) \cos(5y)$

**Problem 2.** Compute  $f_x$ ,  $f_{xx}$ ,  $f_{xy}$ ,  $f_y$ ,  $f_{yy}$ , and  $f_{yx}$  for each function below.

a.  $f(x, y) = y^2e^{3x}$

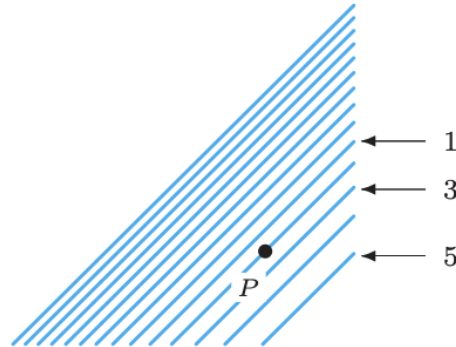
b.  $f(x, y) = e^{x^2+y^3}$

**Problem 3.** The figure below shows the graph of a function  $f(x, y)$ . The points  $(1, 2, f(1, 2))$  and  $(-1, 2, f(-1, 2))$  on its graph are each marked with a red dot. Determine the signs (positive, negative, or zero) of each partial derivative listed below and fill in the table.



Derivative	Sign
$f_x(1, 2)$	
$f_y(1, 2)$	
$f_{xy}(1, 2)$	
$f_x(-1, 2)$	
$f_y(-1, 2)$	
$f_{xx}(-1, 2)$	
$f_{yy}(-1, 2)$	
$f_{xy}(-1, 2)$	

**Problem 4.** The figure below shows a contour plot of  $f(x, y)$ . Determine the signs (positive, negative, or zero) of each partial derivative listed below and fill in the table.



Derivative	Sign
$f_x(P)$	
$f_y(P)$	
$f_{xx}(P)$	
$f_{yy}(P)$	
$f_{xy}(P)$	

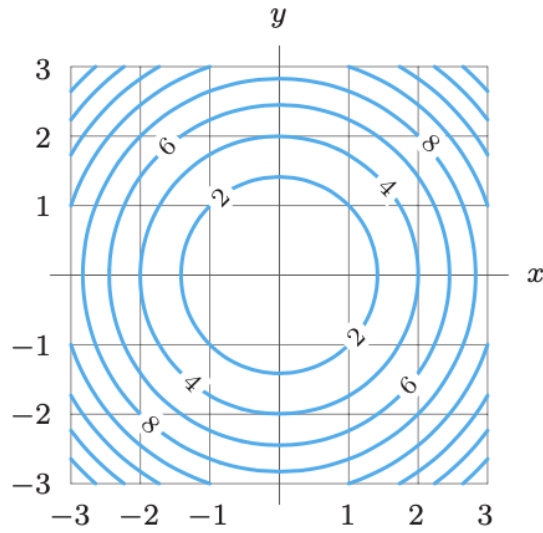
**Problem 5.** Let  $f(x, y) = x^2y^3 - 2x$  and  $P = (1, 1)$ . Compute  $D_{\mathbf{u}}f(P)$  for each unit vector  $\mathbf{u}$  given below.

- $\mathbf{u}$  in the direction of  $\mathbf{v} = \langle 3, 3 \rangle$
- $\mathbf{u}$  in the direction from  $P$  to  $Q = (1, 2)$
- $\mathbf{u}$  in the direction of maximum rate of change
- $\mathbf{u}$  in the direction of minimum (ie. most negative) rate of change
- $\mathbf{u}$  in the direction perpendicular to  $\nabla f(P)$

**Problem 6.** Let  $f(x, y) = \sin x \cos y$  and  $P = (\pi/4, \pi/3)$ . Compute  $D_{\mathbf{u}}f(P)$  for each unit vector  $\mathbf{u}$  given below.

- $\mathbf{u}$  in the direction of  $\mathbf{v} = \langle 1, 1 \rangle$
- $\mathbf{u}$  in the direction from  $P$  to  $Q = (0, 0)$
- $\mathbf{u}$  in the direction of maximum rate of change
- $\mathbf{u}$  in the direction of minimum (ie. most negative) rate of change
- $\mathbf{u}$  in the direction perpendicular to  $\nabla f(P)$

**Problem 7.** Consider the function  $f$  whose contour plot is shown below. Determine the sign (positive, negative, or zero) of  $D_{\mathbf{u}}f(P)$  for each choice of point  $P$  and unit vector  $\mathbf{u}$  in the direction of  $\mathbf{v}$  given below.



Derivative	Sign
$\mathbf{v} = \langle -1, 1 \rangle, P = (-2, 2)$	
$\mathbf{v} = \langle 1, -1 \rangle, P = (-2, 2)$	
$\mathbf{v} = \langle 1, 1 \rangle, P = (-2, 0)$	
$\mathbf{v} = \langle 0, 1 \rangle, P = (-2, 0)$	
$\mathbf{v} = \langle 1, 1 \rangle, P = (0, -2)$	
$\mathbf{v} = \langle -1, 0 \rangle, P = (0, -2)$	

**Problem 8.** Consider the function  $f$  whose contour plot is shown four times below. In the first copy of the contour plot, sketch a vector with basepoint  $(4, 1)$  in the direction of  $\nabla f(4, 1)$ . In the second copy, sketch a vector with basepoint  $(4, 1)$  in a direction  $\mathbf{u}$  such that  $D_{\mathbf{u}}f(4, 1) = 0$ . In the third copy, sketch a vector with basepoint  $(5, 2)$  in the direction of  $-\nabla f(5, 2)$ . In the fourth copy, sketch a vector with basepoint  $(5, 2)$  in the direction  $\mathbf{u}$  such that  $D_{\mathbf{u}}f(5, 2) = 0$ .

