# Math 203, Spring 2023 — Homework 6

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Due March 24

**Instructions.** This problem set has material from Week 7 of class.

**Problem 1.** Evaluate the following integrals.

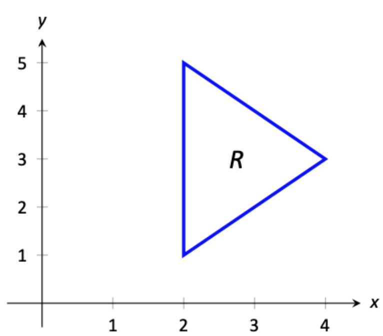
a. $\int_0^3 \int_0^4 (4x + 3y)\,dxdy$

b. $\int_0^1 \int_0^2 x^2 y\,dydx$

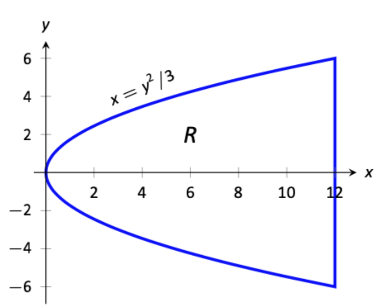
c. $\int_0^3 \int_0^y \sin x\,dxdy$

d. $\int_0^1 \int_0^1 ye^{xy}\,dxdy$

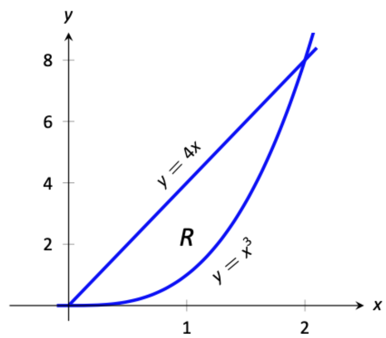
**Problem 2.** For each region $R$ below, give the iterated integrals that represent its area with both orders of integrals, $dydx$ and $dxdy$. No need to evaluate the integrals (but you can use software like Wolfram Alpha to compute them and check that the value is the same with either order of integration).




a.




b.




c.

**Problem 3.** Each iterated integral below represents the area of a region $R$ in the $xy$-plane. Sketch the region $R$ and give the equivalent iterated integral with the opposite order of integration. No need to evaluate the integrals (but you can use software like Wolfram Alpha to compute them and check that the value is the same with either order of integration).

a. $\int_0^1 \int_{5-5x}^{5-5x^2} dydx$

b. $\int_{-2}^2 \int_0^{4-x^2} dydx$

c. $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_1^4 \int_{y-2}^{\sqrt{y}} dxdy$

**Problem 4.** For each double integral below, sketch the region $R$ given by the problem and set up iterated integrals for both orders of integration ($dxdy$ and $dydx$). No need to evaluate the integrals (but you can use software like Wolfram Alpha to compute them and check that the value is the same with either order of integration).

a. $\iint_R ye^x \, dA$ where $R$ is bounded by $x = 0, x = y^2, y = 1$.

b. $\iint_R (6 - 3x - 2y) \, dA$ where $R$ is bounded by $x = 0, y = 0, 3x + 2y = 6$.

c. $\iint_R (x^3 y - x) \, dA$ where $R$ is bounded by the circle $x^2 + y^2 = 9$ in the first and second quadrants.

**Problem 5.** Evaluate each of the following integrals by reversing the order of integration.

a. $\int_0^1 \int_y^1 e^{x^2} \, dxdy$

b. $\int_0^1 \int_y^1 \sin(x^2) \, dxdy$

c. $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2 + x^3} \, dxdy$

**Problem 6.** Let $D$ be the square in the $xy$-plane centered at the origin and given by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. Let $L$ be the left half of this square (ie. $(x, y)$ points where $-1 \leq x \leq 0$ and $-1 \leq y \leq 1$), and let $T$ be the top half of this square (ie. $(x, y)$ points where $-1 \leq x \leq 1$ and $0 \leq y \leq 1$. Determine the sign (positive, negative, or zero) of the following double integrals.

| Integral | Sign |
|---|---|
| $\iint_D x\, dA$ | |
| $\iint_D y\, dA$ | |
| $\iint_D (1-x^2)\, dA$ | |
| $\iint_D (-1+y^2)\, dA$ | |
| $\iint_L x\, dA$ | |
| $\iint_L y\, dA$ | |
| $\iint_T x\, dA$ | |
| $\iint_T y\, dA$ | |

**Problem 7.** For description in polar coordinates below, make a sketch of the given region.

a. $1 \le r \le 2$

b. $\pi/4 \le \theta \le 7\pi/4$

c. $\pi/4 \le \theta \le 5\pi/4$, $r \le 3$

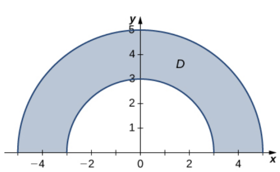
d. $-\pi/2 \le \theta \le \pi/2$, $4 \le r \le 5$

e. $-3\pi/4 \le \theta \le -\pi/4$, $r \le 1$

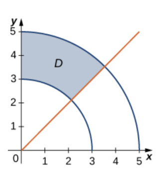
f. $-\pi/2 \le \theta \le 0$, $r \le 2\cos\theta$

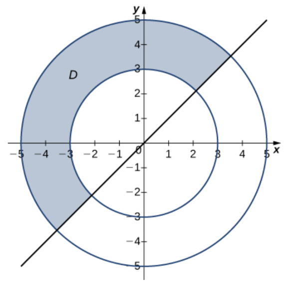
**Problem 8.** For each region in the $xy$-plane shown below, describe it using inequalities involving the polar variables $r$ and $\theta$.

a.

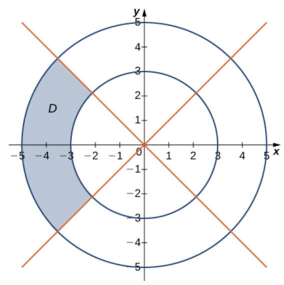



b.






c.




d.