# Math 203, Spring 2023 — Homework 7

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Due April 7

**Instructions.** This problem set has material from Week 9 of class.

**Problem 1.** For each function $f(x, y)$ and region $R$ given below, set up $\iint_R f(x,y)\,dA$ using polar coordinates. No need to compute the integrals.

a. $f(x, y) = 4xy$, $R$ is the portion of the annulus with inner and outer radiuses 1 and 2 in the third and fourth quadrants of the $xy$-plane.

b. $f(x, y) = 1 - x^2 - y^2$, $R$ is the portion of the disk of radius 3 in the first, third, and fourth quadrants of the $xy$-plane.

c. $f(x, y) = x + y$, $R$ is the region between $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$ in the right half of the $xy$-plane.

d. $f(x, y) = 1$, $R$ is the region between the circles $(x-1)^2+y^2 = 1$ and $(x-3)^2+y^2 = 9$.

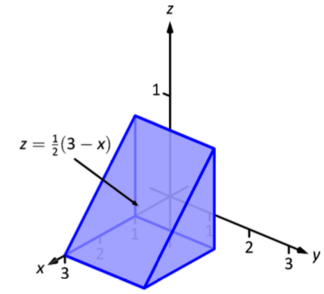
**Problem 2.** For each double integral below, (1) sketch the region of integration, (2) and set up the integral in polar coordinates. No need to compute the integrals.

a. $\int_0^{\sqrt{2}/2} \int_{-\sqrt{1-y^2}}^{-y} (2x + y)\,dxdy$

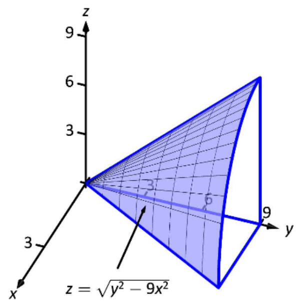
b. $\int_{-\sqrt{2}/2}^{\sqrt{2}/2} \int_{-x}^{\sqrt{1-x^2}} (x + y)\,dydx + \int_{\sqrt{2}/2}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (x + y)\,dydx$

c. $\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x\,dydx + \int_1^2 \int_0^{\sqrt{4-x^2}} x\,dydx.$

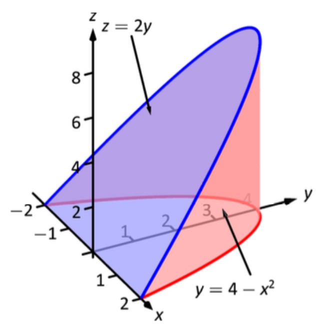
**Problem 3.** Let $D$ be the region shown below. Set up the triple integral $\iiint_D dV$ for the volume of $D$ in three ways: using $dV$ given by (1) $dzdydx$, (2) $dxdzdy$, and (3) $dydzdx$.



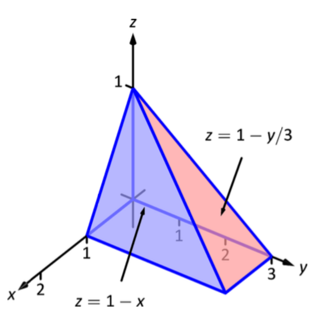

**Problem 4.** Let $D$ be the region shown below. Set up the triple integral $\iiint_D dV$ for the volume of $D$ in three ways: using $dV$ given by (1) $dzdydx$, (2) $dxdzdy$, and (3) $dydzdx$.




**Problem 5.** Let $D$ be the region shown below. Set up the triple integral $\iiint_D dV$ for the volume of $D$ in three ways: using $dV$ given by (1) $dzdydx$, (2) $dxdzdy$, and (3) $dydzdx$.




**Problem 6.** Let $D$ be the region shown below. Set up the triple integral $\iiint_D dV$ for the volume of $D$ in three ways: using $dV$ given by (1) $dzdydx$, (2) $dxdzdy$, and (3) $dydzdx$.



**Problem 7.** For each solid $D$ and corresponding density $f$ described below, set up a triple integral $\iiint_D f(r, \theta, z)\, dV$ in cylindrical coordinates to find the mass of the solid.

a. $f(x, y, z) = z$, $D$ bounded by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ and between the planes $z = 0$ and $z = 10$.

b. $f(x, y, z) = \sqrt{x^2 + y^2} + 1$, $D$ given by the upper half of the unit ball centered at the origin.

c. $f(x, y, z) = x + y$, $D$ bounded by the cone $z = 4 - \sqrt{x^2 + y^2}$ and the plane $z = 0$.

**Problem 8.** For each triple integral below, given in cylindrical coordinates make a sketch of the region of integration and give a brief description of the region in words.

a. $\int_0^{2\pi} \int_3^4 \int_0^5 r\, dz dr d\theta$

b. $\int_0^{\pi} \int_0^1 \int_0^{2-r} r\, dz dr d\theta$

c. $\int_0^{2\pi} \int_0^a \int_0^{\sqrt{a^2 - r^2} + b} r\, dz dr d\theta$