Math 203, Spring 2023 — Homework 8

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Due April 14

Instructions. This problem set has material from Week 11 of class.

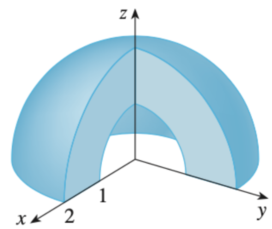
Problem 1. For each point (ρ, θ, ϕ) given in spherical coordinates below, identify the sign of each component of its Cartesian coordinates (x, y, z). For example, if the point has positive x, negative y, and $z = 0$ your answer should be $(+, -, 0)$.

spherical point	Cartesian signs
$(1, 0, 0)$	
$(1, \pi/3, \pi/6)$	
$(1, 2\pi/3, \pi/3)$	
$(1, \pi, \pi/2)$	
$(1, 4\pi/3, 2\pi/3)$	
$(1, 5\pi/3, 5\pi/6)$	
$(1, 2\pi, \pi)$	

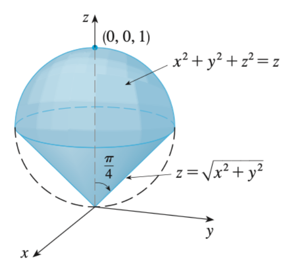
Problem 2. For each solid described below, set up a triple integral in spherical coordinates to find the volume of the solid. No need to compute the integrals.

a. The solid hemisphere of radius R centered at the origin with $z \geq 0$.

b. The solid shown below.



c. The solid shown below.



Problem 3. For each solid and corresponding density described below, set up a triple integral in spherical coordinates to find the mass of the solid. No need to compute the integrals.

a. The solid region where $x^2 + y^2 + z^2 \leq 9$ and $x \leq 0, y \leq 0, z \leq 0$ with density function $f(x, y, z) = z$.

b. The half of spherical shell between the spheres of radius 4 and 5 where $x \leq 0$ with density function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

c. The solid ice cream cone bounded between $x = \sqrt{y^2 + z^2}$ and $x^2 + y^2 + z^2 = 1$ with density function $f(x, y, z) = x$.

Problem 4. Set up a line integral for $\int_C f\, ds$ for each given curve C and function f below. Use Wolfram Alpha to find the numerical value of the integral.

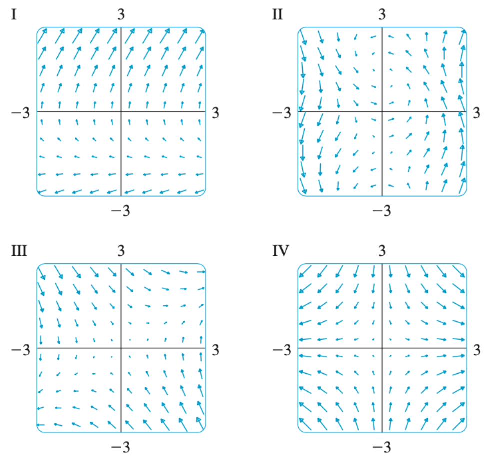
a. $f(x, y) = 5x + 2y$; C is the segment of $y = 3x + 2$ on $[1, 2]$

b. $f(x, y) = y$; C is the curve given by the right half of the unit circle.

c. $f(x, y) = x^2 + y^2$; C is piecewise defined by the line segment connecting $(0, 0)$ to $(1, 1)$ and the line segment connecting $(1, 1)$ to $(1, -3)$.

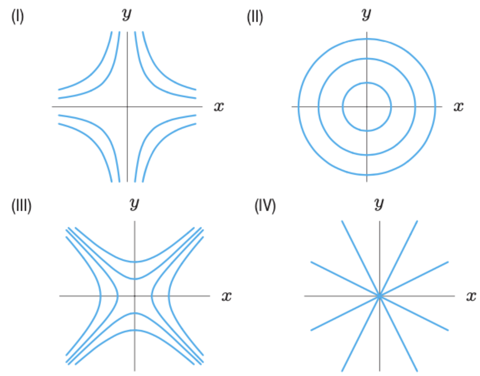
d. $f(x, y) = 2x + 3y + 5$; C is piecewise defined by the curve $y = x^3$ from $(0, 0)$ to $(2, 8)$ and the line segment from $(0, 0)$ to $(2, 8)$.

Problem 5. Match the vector fields $\mathbf{F}$ with the plots labeled I-IV.



Vector field	Plot
$\mathbf{F}(x,y) = \langle x, -y \rangle$	
$\mathbf{F}(x,y) = \langle y, x - y \rangle$	
$\mathbf{F}(x,y) = \langle y, y + 2 \rangle$	
$\mathbf{F}(x,y) = \langle \cos(x + y), x \rangle$	

Problem 6. Match the vector fields **F** with the plots of corresponding flow lines labeled I-IV.



Vector field	Plot
$\mathbf{F}(x, y) = \langle x, y \rangle$	
$\mathbf{F}(x, y) = \langle x, -y \rangle$	
$\mathbf{F}(x, y) = \langle y, -x \rangle$	
$\mathbf{F}(x, y) = \langle y, x \rangle$	

Problem 7. Match the vector fields **F** with the corresponding flow lines described by the parametric curves given below. Begin by using Calplot 3d to sketch the vector fields to make a guess of which the correct flow line. Check your guess by checking the equation $\mathbf{r}'(t) = \mathbf{F}(\mathbf{r}(t))$.

a. $\mathbf{r}(t) = \langle t, \frac{1}{2}t^2 \rangle$

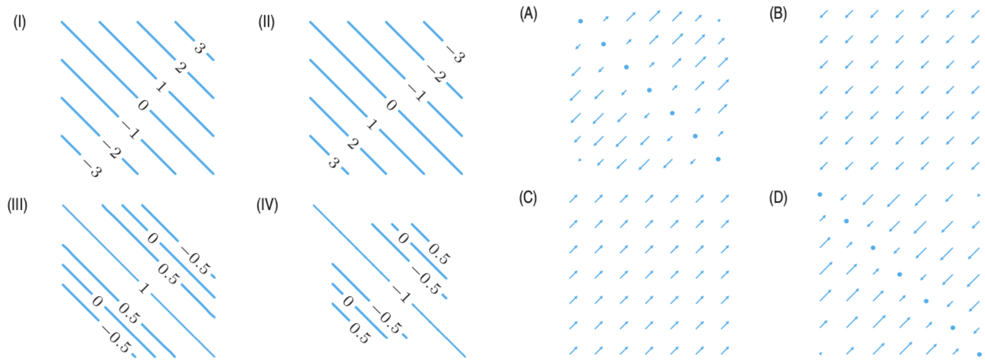
b. $\mathbf{r}(t) = \left\langle \frac{1}{2}t^2, t \right\rangle$

c. $\mathbf{r}(t) = \langle 2\cos t, \sin t \rangle$

d. $\mathbf{r}(t) = \langle e^t, e^{-t} \rangle$

Vector field	Flow line
$\mathbf{F}(x, y) = \langle y, 1 \rangle$	
$\mathbf{F}(x, y) = \langle x, -y \rangle$	
$\mathbf{F}(x, y) = \langle 1, x \rangle$	
$\mathbf{F}(x, y) = \langle -2y, x \rangle$	

Problem 8. Match the contour plots for functions f labeled I-IV with the corresponding vector fields $\mathbf{F}$ labeled A-D so that $\mathbf{F}$ is a gradient vector field with potential function f.



Contour plot	Plot
I	
II	
III	
IV	