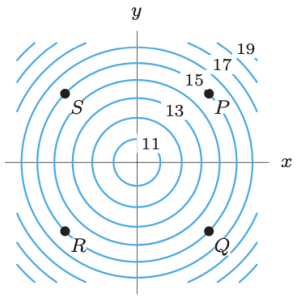
Math 203 — Directional derivatives

Problem 1. In the contour plot below sketch the direction of ∇f at each of the points P, Q, R, and S. Also sketch a direction $\mathbf{u}$ where $D_{\mathbf{u}}f$ is zero at each of these points.



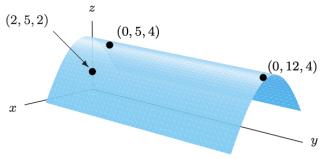
Problem 2. Consider the graph of a function $f(x, y)$ shown below. Give sign of the following directional derivatives

a. $D_{\mathbf{u}}f(2, 5)$ where $\mathbf{u} = \langle -1, 0\rangle$

b. $D_{\mathbf{u}}f(2, 5)$ where $\mathbf{u} = \langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$

c. $D_{\mathbf{u}}f(0, 5)$ where $\mathbf{u} = \langle 0, 1\rangle$

d. $D_{\mathbf{u}}f(0, 12)$ where $\mathbf{u} = \langle 1/\sqrt{2}, -1/\sqrt{2}\rangle$



Problem 3. Let $f(x, y) = -x^2y + xy^2 + xy$ and $P = (2, 1)$. Compute $D_{\mathbf{u}}f(P)$ for each unit vector $\mathbf{u}$ given below.

a. $\mathbf{u}$ in the direction of $\mathbf{v} = \langle 3, 4\rangle$

b. $\mathbf{u}$ in the direction from P to $Q = (1, -1)$

c. $\mathbf{u}$ in the direction of maximum rate of change

d. $\mathbf{u}$ in the direction of minimum (ie. most negative) rate of change

e. $\mathbf{u}$ in the direction perpendicular to $\nabla f(P)$

Problem 4. Repeat the previous problem using $f(x, y) = x^2 + 2y^2 - xy - 7x$ and $P = (1, 1)$.