# Math 203 — Tangent planes

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**Problem 1.** At a distance $x$ feet from the beach, the price in thousands of dollars of a plot of land with area $a$ square feet is given by $f(a, x)$. Suppose $f(1000, 300) = 200$, $f_a(1000, 300) = 3$, and $f_x(1000, 300) = -2$.

a. What are the units and practical meaning of $f(1000, 300)$?

b. What are the units and practical meaning of $f_a(1000, 300)$?

c. What are the units and practical meaning of $f_x(1000, 300)$?

d. Which is cheaper: 1005 square feet that are 305 feet from the beach or 998 square feet that are 295 feet from the beach? Justify your answer with tangent plane approximations.

**Problem 2.** The volume $V$ of a certain gas is related to its temperature $T$ and pressure $P$ by the function $V = f(T, P)$. Suppose volume (in cubic inches) at temperature 500 degrees and pressure 24 (in pounds per square inch) is 23.69 cubic inches. If the pressure increases to 26 pounds per square inch and the temperature is held fixed at 500, the volume becomes 21.86 cubic inches. If the temperature increases to 520 and the pressure is held fixed at 24 pounds per square inch, the volume becomes 24.20 cubic inches.

a. Estimate $f_T(500, 24)$ and $f_P(500, 24)$ using the given information.

b. Estimate the value of $f(505, 24.3)$ using a tangent plane approximation of $f$.

**Problem 3.** For each function $f$ and point $(x_0, y_0)$ below, find the equation of the tangent plane at $(x_0, y_0, f(x_0, y_0))$.

a. $f(x, y) = ye^{xy}, (x_0, y_0) = (1, 1)$

b. $f(x, y) = \sin(xy), (x_0, y_0) = (2, 3\pi/4)$

c. $f(x, y) = \ln(x^2 + 1) + y^2, (x_0, y_0) = (0, 3)$