Math 203 — Exam 1 review

Your exam in class on February 24 will contain about 7 multi-part problems. It will cover material from Homework 0 to Homework 3 (in the textbook, this is material spanning sections 10.1 through 12.1, though we have skipped some sections in that range and these skipped sections will not be part of the exam). The problems below give you a sampling of some similar problems, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet (eg. the orthogonal projection formula) as well as the unit circle with corresponding values of sine and cosine. You'll be allowed to use a scientific calculator with no graphing or calculus functionality but the problems will be written so that a calculator is not required.

Problem 1. Consider the vectors $\mathbf{v} = \langle 3, 2, -2 \rangle$ and $\mathbf{w} = \langle 4, -3, 1 \rangle$.

a. Compute $\mathbf{v} \cdot \mathbf{w}$ and $\mathbf{v} \times \mathbf{w}$.

b. Find the cosine of the angle θ between $\mathbf{v}$ and $\mathbf{w}$.

c. Which option is true: (i) $\theta = 0$, (ii) $0 < \theta < \pi/2$, (iii) $\theta = \pi/2$, (iv) $\pi/2 < \theta < \pi$, (v) $\theta = \pi$?

d. Find the unit vector in the direction opposite of $\mathbf{v}$.

e. Find a vector of length 5 in the direction of $\mathbf{w}$.

f. Make a general sketch, not specific to these vectors, of a parallelogram using $\mathbf{v}$ and $\mathbf{w}$ and label the two diagonals of the parallelogram with $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$ appropriately.

g. Find the area of the parallelogram formed by $\mathbf{v}$ and $\mathbf{w}$.

h. Find the area of the triangle formed by the vectors $\mathbf{v}, \mathbf{w}, \mathbf{v} - \mathbf{w}$.

i. Make a general sketch, not specific to these vectors, of $\mathbf{v}$, $\mathbf{w}$, and the orthogonal projection of $\mathbf{v}$ onto $\mathbf{w}$.

j. Find the orthogonal projection $\mathbf{p}$ of $\mathbf{v}$ onto $\mathbf{w}$.

k. Suppose $\mathbf{q} = \mathbf{v} - \mathbf{p}$. Find $\mathbf{q} \cdot \mathbf{w}$.

l. Find a standard form equation of the plane containing $P = (1, 2, 3)$ and parallel to both $\mathbf{v}$ and $\mathbf{w}$.

m. Find a vector equation of the line containing P and orthogonal to the plane in the previous part.

Problem 2. Consider the line ℓ_1 passing through the point $(1,2,4)$ in the direction of $\langle 3,1,-1\rangle$, the line $\ell_2(t) = \langle 1,1,3\rangle + t\langle 3,1,2\rangle$, the plane Π_1 with equation $5(x-1)+2y-3(z+1)=0$ and the plane Π_2 with equation $3x+y+2z=10$.

a. Determine whether ℓ_1 and ℓ_2 are parallel, intersect, or are skew lines.

b. Determine whether ℓ_1 is orthogonal to Π_1. Do the same with Π_2.

c. Repeat the previous question with ℓ_2.

d. Determine whether ℓ_1 is parallel to Π_1. Do the same with Π_2.

e. Repeat the previous question with ℓ_2.

f. If ℓ_1 intersects Π_1 find the point of intersection. Do the same with Π_2.

g. Repeat the previous question with ℓ_2.

h. Determine whether Π_1 and Π_2 are parallel.

Problem 3. Sketch the surfaces determine by the following equations and give the technical term for the shape (eg. *circular cylinder*).

a. $z = 4-(x^2+y^2)$

b. $z = 1+\sqrt{(x-1)^2+y^2}$

c. $z = x^2$

d. $y^2+z^2=1$

e. $x+y+z=1$

f. $x^2+y^2+z^2=4$

Problem 4. Consider the function $f(x,y) = 4-(x-1)^2-(y-2)^2$.

a. What are the domain and range of f?

b. Make a contour diagram using the $z = 0,1,2,3,4$ level curves.

c. Explain why the contour diagram cannot have any level curves of the form $f(x,y) = c$ where $c > 4$.

d. Sketch the graph of f in three dimensional space, taking note of how it relates to the contour diagram.

Problem 5. Let $\mathbf{r}(t) = \langle 1+3\cos(-t), 2+3\sin(-t)\rangle$ for $0 \le t \le 2\pi$.

a. Describe in words and plot the curve traced by $\mathbf{r}(t)$.

b. Compute $\mathbf{r}'(t)$.

c. Find the tangent vector of $\mathbf{r}(t)$ when $t = \pi/3$ and plot it in your sketch above.

d. Find a vector equation of the tangent line to the curve when $t = \pi/3$.

Problem 6. Compute $\mathbf{r}'(t)$ for each example below.

a. $\mathbf{r}(t) = \left\langle e^{t^2}, \sin(t^3+3t^2), \cos(e^{5t})\right\rangle$

b. $\mathbf{r}(t) = \langle t\ln t, t^2\sin(4t), t^3e^{2t}\rangle$

c. $\mathbf{r}(t) = \left\langle \frac{e^{2t}}{t^3+t^2}, \frac{t^4}{\sin(2t)+\cos t}, \frac{t^5+t^3}{t^2+1}\right\rangle$