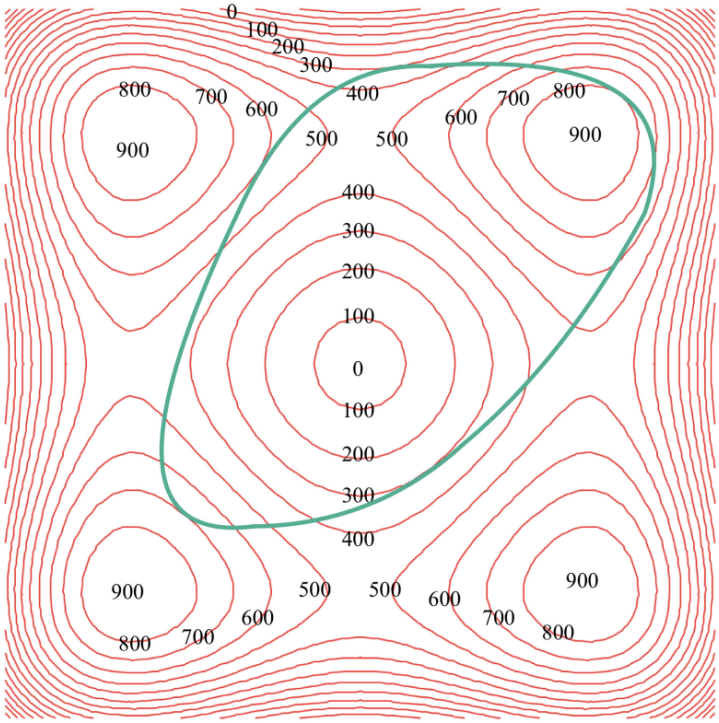
Math 203 — Lagrange Multipliers Warm-up

Imagine that the following contour diagram represents the elevation of a terrain, where each contour line shows points of equal height for the elevation function $f(x, y)$. Suppose the green curve on the diagram represents a hiking trail. It is absolutely necessary that we stay on this trail—stepping off would make us an irresponsible hiker! Mathematically, the trail might be represented by the equation $g(x, y) = c$.



Problem 1. Forget the hiking trail for a moment. On the contour diagram

- Mark with $\wedge$ all the points where f has local maxima
- Mark with $\vee$ all the points where f has local minima.

These are the unconstrained extrema of the elevation function.

Problem 2. Now suppose we're on the hiking trail and must stay on it. On the contour diagram

- Mark with $\times$ each point where the trail levels off after a climb and begins to descend.
- Mark with $\oplus$ each point where the trail levels off after a descent and beings to climb.

These are the constrained extrema of the elevation function.

Problem 3. At each of the points (a, b) marked with $\times$ or $\oplus$ sketch $\nabla f(a, b)$.

Problem 4. We can think of the green curve $g(x, y) = c$ as a level curve of a function $g(x, y)$. As such, we can compute gradient vectors of g and these vectors will be orthogonal to the level curves of g. At each of the points (a, b) marked with $\times$ or $\oplus$, how is $\nabla g(a, b)$ related to $\nabla f(a, b)$?