Math 203 — Line integrals of scalar fields

Problem 1. Set up the line integral $\int_C 1\,ds$ for the arclength of each given curve.

a. C is the portion of $y = x^3$ from $(-1,-1)$ to $(1,1)$.

b. C is the portion of $x = -1 + y^2$ from $(0,-1)$ to $(0,1)$.

c. C is the upper half of the ellipse $\mathbf{r}(t) = \langle 2\cos t, \sin t\rangle$.

Problem 2. Set up the line integral $\int_C f\,ds$ for each given plane curve C and surface $f(x,y)$. Use software like Wolfram Alpha to compute the integral.

a. C is the parabola $x = y^2$ oriented from $(4,-2)$ to $(4,2)$; $f(x,y) = \sin x \cos y$

b. C is the line segment joining $(-2,-1)$ and $(1,2)$; $f(x,y) = x^2 + y^2 + 2$

c. C is the right half of the unit circle along with the line segment from $(0,1)$ to $(0,-1)$; $f(x,y) = x + y$

d. C is the line segments from $(0,1)$ to $(1,1)$ and from $(1,1)$ to $(1,0)$; $f(x,y) = x + y^2$

Problem 3. Let C be the piecewise defined closed curve in the shape of a square with vertices at $(\pm 1, \pm 1)$. Let C_L, C_R, C_T, C_B denote the four line segments that form C (left, right, top, bottom respectively). Give the sign (positive, negative, or zero) of each of the following line integrals.

a. $\int_{C_R} x\,ds$

b. $\int_{C_L} x\,ds$

c. $\int_{C_T} x\,ds$

d. $\int_{C_B} x\,ds$

e. $\int_C x\,ds$

f. $\int_C (x^2 + y^2)\,ds$