# Math 203 — Path independence and conservative vector fields

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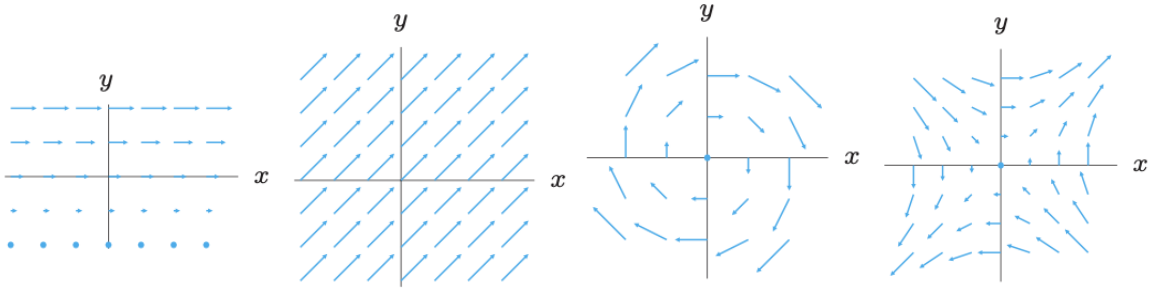
**Problem 1.** Let $C$ be the closed curve consisting of a square centered at the origin with vertices at $(\pm1, \pm1)$, oriented counter-clockwise. Determine the sign (positive, negative, or zero) of $\oint_C \mathbf{F} \cdot d\mathbf{r}$ for the vector fields $\mathbf{F}$ shown below. Indicate whether the vector field is path-independent.



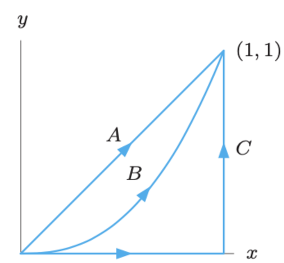

**Problem 2.** For each of the following vector fields $\mathbf{F}$, find a potential function $f$ of $\mathbf{F}$ and use the Fundamental Theorem of Line Integrals to compute $\int_A \mathbf{F} \cdot d\mathbf{r}$, $\int_B \mathbf{F} \cdot d\mathbf{r}$, and $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $A$, $B$, and $C$ are the curves shown below.

a. $\mathbf{F}(x, y) = \langle x, y \rangle$

b. $\mathbf{F}(x, y) = \langle y \cos x, \sin x + y \rangle$

c. $\mathbf{F}(x, y) = \langle 2xy, x^2 \rangle$

d. $\mathbf{F}(x, y) = \langle 2xy, x^2 + 8y^3 \rangle$




**Problem 3.** Try finding a potential function for $\mathbf{F}(x, y) = \langle -y, x \rangle$. What goes wrong?

**Problem 4.** Compute $\int_A \mathbf{F} \cdot d\mathbf{r}$ and $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x, y) = \langle -y, x \rangle$ using the curves $A$ and $C$ from Problem 2. Is $\mathbf{F}$ path-independent?