Math 203 — Divergence Theorem

Problem 1. Let $\mathbf{F} = \langle x - y, x + y \rangle$, let C be the circle of radius 2 centered at the origin, and let R be the region enclosed by C. Verify the Divergence Theorem for this example. That is, compute $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$ using a parametrization and compute $\iint_R \operatorname{div} F\, dA$.

Problem 2. Try Problem 1 using C as the closed, positively oriented curve comprising the parabola $y = x^2$ for $0 \leq x \leq 2$ and the line segment from $(2, 4)$ to $(0, 0)$.

Problem 3. Let $\mathbf{F}$ be a vector field whose domain is all of $\mathbb{R}^2$ with the property that $\operatorname{div} \mathbf{F} = 0$ and let C_1 and C_2 be two non-intersecting curves each oriented so that they start from $(0, 0)$ and go to $(1, 1)$. Suppose $\int_{C_1} \mathbf{F} \cdot \mathbf{n}\, ds = 5$. Find the value of $\int_{C_2} \mathbf{F} \cdot \mathbf{n}\, ds$ and explain your reasoning.